

Optimal Participant Behaviour in an Electricity Market

A.B.Philpott E.J. Anderson*

R.J. Kaye[†]

Department of Engineering Science

University of Auckland

New Zealand

a.philpott@auckland.ac.nz

Abstract

We consider strategies for a generator in a wholesale electricity market in which the generator has some degree of market power. The generator seeks a supply function to offer to the spot market, so as to maximise his expected return. We give conditions under which a supply function exists that optimises return for every demand realisation. The implications of this result for hydro-electric generation are presented. We conclude with an analysis of the case in which the behaviour of the competition can be modelled by an appropriate probability distribution, and derive optimality conditions for the optimal supply-function offer in this case.

1 Introduction

In recent years a number of countries have introduced wholesale electricity spot markets. Although implementations of these markets vary, they all endeavour to deliver power to consumers at a competitive price, and provide sensible signals for investment and new entry. The electricity markets we consider in this paper have the specific feature that the spot price of electricity is determined by solving an optimisation problem which matches supply and demand so as to minimise the total revealed cost of power delivery. The first electricity market to be created of this type was for England and Wales, but similar markets now operate in Norway, Australia and New Zealand.

To determine the spot price of electricity in these markets, the supply of power in a given time interval (typically 30 minutes long) is defined by generators of power in the form of *offers* of blocks of energy at prices chosen by them; together the set of price-quantity pairs is called the *offer stack* for the generator in that time interval.

*Australian Graduate School of Management, University of NSW

[†]Department of Electrical Engineering, University of NSW

The number of blocks for each generator is typically about ten. The spot-market clearing model computes a supply of power to the demands at the nodes of the transmission network, by successively dispatching generation from the offers with lowest price, taking into account any losses and constraints in the transmission network. The final spot prices for energy at the nodes of the transmission network in each time interval are defined by the *marginal values* of the energy balance constraints at the nodes of this optimisation problem. These marginal values are a function of the availability of generation and transmission capacity, the demand for power at the time, and the particular offers made by the generators.

In this paper we consider the problem facing a generator in a given time interval, of constructing an offer stack that maximises the return in this interval in the presence of uncertain demand. Apart from a small number of papers (see e.g. Gross and Finlay, 1996) previous analysis of electricity spot markets has concentrated more on the issue of market performance than on optimal participant behaviour. One important strand of work is that of Green and Newbery (1992, 1995, 1996) who have looked at the performance of the England and Wales market, in which each generator offers a single offer stack a day ahead to operate for each hour of the following day. These authors view generators who actually offer a range of price-quantity bids as instead (approximately) offering a continuous “supply function”. As shown by Klemperer and Meyer (1989) under certain conditions supply-function offers can be constructed for each market participant in an oligopoly, which give a Nash equilibrium even when demand is uncertain.

In contrast to the England and Wales market, the offer stacks in the New Zealand electricity market may vary with each half hour of the day, and in Australia there are opportunities to alter the quantities offered (though not the prices) for any five minute interval. Even though this reduces the variation in demand, which may give one reason to discard a supply-function model, there is still some demand uncertainty since the offer must still be submitted in advance of the time of dispatch.

In contrast to the work of Klemperer and Meyer, we focus on the situation, characteristic of electricity markets in the short term, in which demand is unaltered by price. Of course this assumption of completely inelastic demand is unrealistic over the medium term, but our discussion is aimed at generators who need to decide on an offer stack for today. In practice generators will also hold contract positions which might affect their offering strategies. Although these can be included in the analysis with few essential changes (see Anderson and Philpott 1998a) we have chosen to omit them here for simplicity.

The remainder of the paper is laid out as follows. In the next section we define the notion of a supply function, and give conditions under which a supply function will optimise return irrespective of demand. Section 3 examines the use of supply functions in the context of hydro-electric generation. Here the offers must take into account the value of water remaining in the reservoir after dispatch. The final section derives some optimality conditions for an optimal supply function, when the probability distribution of demand and competitor behaviour is known. Throughout the paper we focus only on the main results. Proofs of these and a more thorough analysis of supply-function offering may be found in the papers by Anderson and Philpott (1998a,1998b,1998c).

2 Supply functions

Consider a generator which is allowed to make k offers, $(p_1, q_1), (p_2, q_2), \dots, (p_k, q_k)$ with $p_1 < p_2 < \dots < p_k$. If the market clears at price p with $p_j < p < p_{j+1}$ then the generator will be asked to dispatch all the offers at a lower price than p , i.e. a total amount of power $\sum_{i=1}^j q_i$, and will be paid at the price p for all the power generated.

The offer stack, consisting of the set of price-quantity pairs, can be modelled by a step function giving the total power dispatched from the generator for each value of clearing price. It is convenient to model an offer stack as a (parameterised) continuous *supply curve* $\mathbf{s} = \{(q(t), p(t)), 0 \leq t \leq T\}$, in which the components $q(t)$ and $p(t)$ are monotonic increasing piecewise differentiable functions of t . The maximum generation capacity of the generator is given by q_M . Without loss of generality we assume that $q(0) = p(0) = 0$, $q(T) = q_M$, and there are no intervals on which $q'(t)$ and $p'(t)$ are both identically zero. Furthermore we require $q'(t)$ and $p'(t)$ to be left continuous. If $p(t)$ is strictly monotonic then the offer stack can be represented by a continuous *supply function* denoted by $S(p)$. When it exists, it is often convenient to work with the inverse of $S(p)$, which we denote by $T(q)$. This gives the price at which a generator offers the amount q .

We will consider the situation of two generators (say A and B) at a single node offering in supply to satisfy an unknown demand h . We first consider the optimal response of generator A to a revealed supply-function offer by generator B for a single known value of h . The second generator may be an amalgam of more than one other generator in the market. We assume that generation of an amount q by A incurs a total cost of $C(q)$, $q \in [0, q_M]$, that we assume is a continuous and increasing function.

Suppose then that generator B offers in a supply function denoted by $S(p)$, $p \in [0, p_M]$. At the clearing price p , the amount of generation required from generator A is $h - S(p)$ for which he makes a return of

$$R = p(h - S(p)) - C(h - S(p)). \quad (1)$$

In what follows we will discuss optimal generator behaviour in the case where h is unknown. In this situation the special case where both $S(\cdot)$ and $C(\cdot)$ are convex increasing functions is important. The condition that $S(\cdot)$ is a convex increasing function means that it is strictly increasing on $[a, p_M]$, where $a = \max\{p | S(p) = S(0)\}$. This implies that the inverse function $T(\cdot) = S^{-1}(\cdot) : [S(0), S(p_M)] \rightarrow [a, p_M]$ exists and is continuous and concave on $[S(0), S(p_M)]$. The return as a function of the quantity q dispatched by generator A is thus

$$R = qT(h - q) - C(q).$$

It is easy to show that if S is convex and C is convex then $qT(h - q) - C(q)$ is a concave function of q . Thus if $qT(h - q) - C(q)$ is smooth then (in the absence of bounds) it is maximised at the value of q which makes R stationary. This gives the condition

$$-qT'(h - q) + T(h - q) - C'(q) = 0, \quad (2)$$

which may be solved to find the optimal value q^* of q , and clearing price $p^* = T(h - q^*)$. It is possible to extend this analysis to the nonsmooth case (see Anderson and Philpott (1998a)). In this paper we shall confine attention to smooth S and C .

Consider now the situation in which h is uncertain. Under the conditions above we can compute an optimal offer $(p(h), q(h))$ for generator A for each possible realization of h in a certain range. In fact we require $h \in H = [S(0), S(p_M)]$. Any choice of $p(h)$ not exceeding the optimal clearing price $p^*(h)$ will constitute an optimal offer in conjunction with $q(h)$. The curve $\mathbf{s} = \{(p(h), q(h)), h \in H\}$ will define a supply curve if its components $p(h)$ and $q(h)$ are monotonic increasing continuous functions of h . Moreover since for each value of $h \in H$, the offer $(p^*(h), h - S(p^*(h)))$ maximises the return to generator A, then choosing the supply curve \mathbf{s} will maximise this generator's expected return, irrespective of the distribution of h . We call a supply curve satisfying these conditions a *supply-curve response*.

We observe that a supply-curve response is not always guaranteed to exist, as shown by the following example.

Example A situation in which a supply-curve response does not exist.

Suppose generator B offers the supply function $S(p) = (10p - 10)^{\frac{1}{3}} - 0.5$, and generator A with zero cost and no contracts, seeks optimal price-quantity offers against the curve S for different realizations of demand h . For $h = 3$ we obtain a return $R(p)$ defined by:

$$R(p) = p(3 - (10p - 10)^{\frac{1}{3}} + 0.5)$$

which has a maximum at $p = 1.0125$. So when $h = 3$, A should offer $h - S(p) = 3$ at $p = 1.0125$. For $h = 5$ the return is defined by:

$$R(p) = p(5 - (10p - 10)^{\frac{1}{3}} + 0.5)$$

which has a maximum at $p = 7.2385$. So A should offer $h - S(p) = 1.5339$ at $p = 7.2385$. Since each of these solutions is the unique optimum for the respective demand, it is clear in this case that a supply-curve response to $S(p)$ does not exist.

The example above illustrates one of the central themes of this section, and it is instructive to study its implications in more detail. A generator offering a supply-curve response to the function $S(\cdot)$ is guaranteed an optimal outcome for any realization of demand. To guarantee this in the example generator A must offer an amount 3 at a price 1.0125 or lower to maximise return for demand of 3. If demand is 5 then for an optimal return, the clearing price will be 7.2385. This will imply a complete dispatch of at least the quantity 3 that has been offered at a price not exceeding 1.0125. This is inconsistent with an optimal return for this demand, which can only be obtained by a dispatch of 1.5339. If, on the other hand, the optimal dispatch for A at $h = 5$ were more than 3 units at a clearing price higher than 1.0125, then A could maximise return for both demand realizations.

In order for a supply-curve response to the function $S(\cdot)$ to be constructed, the optimal dispatch and clearing price must be monotonic increasing functions of demand. Conditions which guarantee this are given in the following theorem.

Theorem 1 *If $S(\cdot)$ and $C(\cdot)$ are increasing convex functions with $S(\cdot)$ differentiable, then there is a supply-function response that is optimal for any $h \in H$.*

It is worth observing that the model we have developed is identical to that of a monopolist offering generation to a known decreasing market demand function that is subject to random changes in demand that are unknown in advance. We can simply write the demand curve as $D(p) = h - S(p)$ where changes in h reflect changes in the demand. In this particular set of circumstances, the theorem shows that it is appropriate for the monopolist to offer a supply function into the market which has the effect of picking out the optimal point on the demand curve for a range of possible values of h .

Corollary 2 *Suppose a monopolist faces a base demand function $D(p)$ where $D_p < 0$, $D_{pp} < 0$, but actual demand is $\max(0, D(p) - h)$ where h is an unknown demand loss subtracted from the base demand. If the monopolist has an increasing marginal cost function, has no advance knowledge of the value of h , and plays a single shot game, then there is a choice of supply function which maximises the return to the monopolist for any h in the range $(0, D(0))$.*

We digress briefly here to consider the existence of a *supply-function equilibrium*. This is defined to be a pair of supply functions $(S_A(\cdot), S_B(\cdot))$, with the property that $S_A(\cdot)$ is a supply-function response to $S_B(\cdot)$, and $S_B(\cdot)$ is a supply-function response to $S_A(\cdot)$ (both of these properties holding for $h \in H$). A *symmetric* supply-function equilibrium has $S_A(\cdot) = S_B(\cdot)$. Supply-function equilibria are just Nash equilibria in the single-shot two-person game between A and B. Such equilibria have been studied extensively by Klemperer and Meyer (1989) in the case where there is a demand elasticity as well as uncertainty.

Consider two identical generators at the same location. Suppose that they both have differentiable convex cost functions $C(\cdot)$ and offer in strictly increasing differentiable convex supply functions $S(\cdot)$, and together they face an uncertain demand h for power in the interval $[0, h_M]$. For a given demand h , a clearing price satisfies $S(p) + S(p) = h$, which shows that each generator will be dispatched half the demand. The first-order optimality conditions give

$$qT'(q) - (T(q) - C'(q)) = 0. \quad (3)$$

Define $\psi(q)$ to be any function satisfying $q^2\psi'(q) = C'(q)$. Then the general solution to (3) is

$$T(q) = kq - q\psi(q),$$

where k is an arbitrary constant. It is a straightforward matter (see Anderson and Philpott, 1998a) to show that k can be chosen so that $T(q)$ is increasing and concave, and that it is an optimal supply-function response to itself. This is summarized in the following theorem.

Theorem 3 *Suppose that two generators have identical differentiable convex cost functions $C(\cdot)$. Let $\psi(q)$ be any function satisfying $q^2\psi'(q) = C'(q)$. Then a supply-function equilibrium exists for the range of demands $H = [0, h_M]$, in which each participant offers $S(\cdot) = T^{-1}(\cdot)$, where $T(q) = kq - q\psi(q)$, and $h_M < q_M$ is chosen so that $k > \frac{C'(h_M)}{h_M} + \psi(h_M)$.*

3 Supply functions for hydro-electric generators

In this section we consider the application of the results above to a hydro-electric generator. One of the features of hydro-electricity which makes it substantially different from other forms of generation is the ability to store energy (in terms of water) for later use. In centrally planned systems, hydro-electric reservoir releases are typically determined using water value surfaces. For any time of the year and for any storage level these give the marginal value of energy stored in the reservoir. If releasing water from the reservoir decreases the cost of supplying the demand by more than the marginal water value then the system should do so. Otherwise the water should be retained. Computing water value surfaces requires the solution to a dynamic optimisation problem, which imputes a water value from the cost of thermal generation that any hydro generation replaces. For one or two reservoirs this optimisation can be carried out using dynamic programming (see e.g. Yang and Read, 1990). For many reservoirs, some form of stochastic Benders decomposition is used (see e.g. Jacobs *et al*, 1995).

In a market context, the cost of hydro generation can no longer be viewed as the expected saving from avoiding future thermal generation, since thermal sources may not be owned by the hydro participant. Furthermore, even when a generator owns a combination of hydro and thermal stations, it is not clear that minimising the cost of power delivery is an appropriate objective when the generator has the ability to influence prices by their policy. Instead the cost of generation by releasing some water should be viewed as the (expected present value of the) revenue in later periods which is foregone, given that we follow an optimal policy now and in the future.

In practice the optimal policy at each stage will consist of a set of offers given by a step function. The advantage of offering a function rather than a single tranche comes from the ability to optimise over a range of unknown demands. As shown above we can guarantee this advantage when the cost of generation is convex and the (aggregated) supply function of other generators is smooth and convex. Otherwise it seems to be necessary to take account of the distribution of demand when constructing offer stacks.

Let $V_t(x)$ be the (present) value of the expected revenue stream that accrues from operating the reservoir optimally with t time steps remaining to the end of the planning horizon, given that there is x units of energy in the reservoir. (We choose $V_0(x) = 0$, for every x .) If we release q in the interval between t and $t - 1$ then at $t - 1$ we shall have $x + i - q$ where i is a random inflow. Thus the expected revenue stream that accrues in later periods is $\alpha E_i[V_{t-1}(x + i - q)]$, where α is the one-period discount factor. Alternatively, for any x , $C_x(q) = -\alpha E_i[V_{t-1}(x + i - q)]$ can be viewed as the (future) cost of releasing q .

Let the (random) demand in the interval between t and $t - 1$ be denoted by h_t , and suppose the rest of the generators in aggregate offer a curve $T(q)$, which gives the minimum price at which they would generate q . To maximise revenue in the interval $[t - 1, t]$ the hydro-electric generator seeks to maximise $qT(h_t - q) - C(q)$. If C is convex and T is concave (corresponding to a convex supply function) then for each h_t the optimal offer maximising $qT(h_t - q) - C(q)$ gives rise to a supply function response to T . The expected revenue from this optimal response is given

by the following recursion.

$$V_t(x) = E_{h_t} \max_{q \in [0, x]} \{qT(h_t - q) + \alpha E_i[V_{t-1}(x + i - q)]\}. \quad (4)$$

In order to apply (4) we require that a supply-function response exists at each stage. If T is concave then a sufficient condition for a supply-function response to exist is to have $\alpha E_i[V_{t-1}(x + i - q)]$ a concave function of q for each x . In fact it can be shown (see Anderson and Philpott, 1998b) that if $\alpha E_i[V_{t-1}(x + i - q)]$ is concave and increasing then so is $E_{h_t} \max_{q \in [0, x]} \{qT(h_t - q) + \alpha E_i[V_{t-1}(x + i - q)]\}$. Thus the desirable properties of $V_{t-1}(x + i - q)$ (namely concavity and monotonicity) are inherited in V_t every time the recursion is applied. From this observation we can derive the following theorem.

Theorem 4 *If at each time t over a planning horizon $t = 0, 1, \dots, N$, the competition offer stack T_t is an increasing concave revealed function, then at each time stage t there is a supply function response to T_t that maximises the discounted expected return over the planning horizon.*

Observe that this theorem requires perfect knowledge of the competitors' offer stack both now and for all t in the future, and so its practical application is limited. In practice the competitor's offer stack will be unrevealed, even in the current time period, and so agents must attempt to optimise their offers subject to this uncertainty. This issue is addressed in the next section.

4 Maximising expected return

In this section we consider a problem setting in which supply function responses do not exist, and in which we need to take explicit account of the distribution of demand. Since uncertainty of demand also interacts with the offers of other market participants, our analysis will also deal with explicit uncertainty in respect to other offers in the market. This analysis will also be useful in constructing a theory involving mixed strategies in an equilibrium.

Consider then a generator offering in a curve defined by \mathbf{s} . Suppose the aggregated offers of the competitors can be described by the (random) supply function $S(p)$. Let

$$\psi(q, p) = \Pr(q > h - S(p)).$$

Now consider a generator offering in a curve defined by \mathbf{s} , and suppose that \mathbf{s} passes through (q, p) and $(q + \delta q, p + \delta p)$. Then the clearing price p^* will be between p and $p + \delta p$, and the generator will be dispatched (at q^* , say) between q and $q + \delta q$ if $q + S(p) < h$, and $h < q + \delta q + S(p + \delta p)$. Thus

$$\begin{aligned} \Pr(q &\leq q^* < q + \delta q, p \leq p^* < p + \delta p) \\ &= \Pr(q + S(p) \leq h < q + \delta q + S(p + \delta p)) \\ &= \Pr(q + S(p) \leq h < q + \delta q + S(p + \delta p)) \\ &= \Pr(h < q + \delta q + S(p + \delta p)) - \Pr(h < q + S(p)) \\ &= \psi(q + \delta q, p + \delta p) - \psi(q, p) \end{aligned}$$

Suppose that we earn a return $R(q, p)$ if dispatched q at a market price p . The expected return from the supply function offer defined by the curve \mathbf{s} is then defined by the line integral

$$E[R] = \int_{\mathbf{s}} R(q, p) d\psi(q, p).$$

When $\psi(q, p)$ is differentiable, we obtain

$$E[R] = \int_0^T R(q, p) (\nabla \psi)^\top \mathbf{s}'(t) dt = \int_0^T R(q, p) \left[\frac{\partial \psi}{\partial q} q'(t) + \frac{\partial \psi}{\partial p} p'(t) \right] dt.$$

We can obtain optimality conditions for any candidate offer curve by computing the line integral along a feasible variation and comparing it with the original line integral. The difference between the two integrals is a line integral around a closed curve \mathcal{C} enclosing some region \mathcal{S} of the plane. Formally, let

$$Q(q, p) = R(q, p) \frac{\partial \psi}{\partial q},$$

$$P(q, p) = R(q, p) \frac{\partial \psi}{\partial p}.$$

Then

$$\begin{aligned} \frac{\partial P}{\partial q} - \frac{\partial Q}{\partial p} &= R(q, p) \frac{\partial^2 \psi}{\partial q \partial p} + \frac{\partial R}{\partial q} \frac{\partial \psi}{\partial p} - R(q, p) \frac{\partial^2 \psi}{\partial p \partial q} - \frac{\partial R}{\partial p} \frac{\partial \psi}{\partial q} \\ &= \frac{\partial R}{\partial q} \frac{\partial \psi}{\partial p} - \frac{\partial R}{\partial p} \frac{\partial \psi}{\partial q}. \end{aligned}$$

Now applying Green's Theorem in the plane we obtain

$$\begin{aligned} \int \int_{\mathcal{S}} \left(\frac{\partial R}{\partial q} \frac{\partial \psi}{\partial p} - \frac{\partial R}{\partial p} \frac{\partial \psi}{\partial q} \right) dp dq &= \int \int_{\mathcal{S}} \left(\frac{\partial P}{\partial q} - \frac{\partial Q}{\partial p} \right) dp dq \\ &= \int_{\mathcal{C}} Q(q, p) dq + P(q, p) dp \\ &= \int_{\mathcal{C}} R(q, p) \frac{\partial \psi}{\partial q} dq + R(q, p) \frac{\partial \psi}{\partial p} dp \\ &= \int_{\mathcal{C}} R(q, p) d\psi(q, p). \end{aligned}$$

We define for each (q, p) the function

$$Z(q, p) = \frac{\partial R}{\partial q} \frac{\partial \psi}{\partial p} - \frac{\partial R}{\partial p} \frac{\partial \psi}{\partial q}.$$

If any curve \mathcal{C} passes through a region \mathcal{S} in which $Z(q, p) > 0$, then the line integral of $Rd\psi$ along the curve may be improved by adding an anticlockwise traversal of a closed contour lying in \mathcal{S} . If \mathcal{C} passes through any region \mathcal{S} in which $Z(q, p) < 0$, then it may be improved by adding a clockwise traversal of a closed contour lying in \mathcal{S} . Of course this improvement is only valid if the components of the new offer stack are increasing.

The observations above can be formalised and used to derive the following necessary optimality conditions for a candidate offer stack.

Theorem 5 Suppose $(q(t), p(t)), t \in [0, T]$ is an optimal supply curve. Let

$$t_0 = \sup\{t \mid q(t) = 0 \text{ or } p(t) = 0\},$$

$$t_1 = \inf\{t \mid q(t) = q_M\},$$

$$w(t) = \int_0^t Z(q(\tau), p(\tau)) \sqrt{q'(\tau)^2 + p'(\tau)^2} d\tau.$$

Then

$$w(t_1) - w(t_0) = 0,$$

$$q'(t)(w(t) - w(t_0)) \leq 0, \quad t_0 \leq t \leq t_1,$$

$$p'(t)(w(t) - w(t_0)) \geq 0, \quad t_0 \leq t \leq t_1.$$

Although apparently complicated, the necessary conditions above greatly reduce the choice of candidates for an optimal solution. In particular, when $Z(q, p) \neq 0$ the optimal offer curve must be either horizontal or vertical, and can only change direction when $w(t) - w(t_0) = 0$. For reasonably smooth $\psi(q, p)$, this will only occur for a small number of values of t , allowing the construction of an effective enumeration algorithm. This is a topic for future research.

5 Acknowledgments

The first author wishes to acknowledge the support of the New Zealand PGSF under FRST Contract UOA403, and the support of AGSM Special Research Grant G1797.

References

- [1] Anderson, E.J. and Philpott, A.B., Using supply functions for offering generation into an electricity market. Working Paper 98-008, Australian Graduate School of Management, University of New South Wales, June 1998.
- [2] Anderson, E.J. and Philpott, A.B., On supply-function offers for hydro-electric generation. In preparation, August 1998.
- [3] Anderson, E.J. and Philpott, A.B., Optimal offer construction in electricity markets. In preparation, August 1998.
- [4] Green, R.J., Increasing competition in the British electricity spot market. *Journal of Industrial Economics*, 44(2), 1996.
- [5] Green, R.J. and Newbery, D.M., Competition in the British electricity spot market. *Journal of Political Economy*, 100(5):929–953, 1992.
- [6] Gross, G. and Finlay, D.J., Optimal bidding strategies in competitive electricity markets. In *Proceedings of the 12th Power Systems Computation Conference*, Dresden, August 1996.

- [7] J. Jacobs, G. Freeman, Grygier J., D. Morton, G. Schultz, K. Staschus, and J. Stedinger. Socrates: A system for scheduling hydroelectric generation under uncertainty. *Annals of Operations Research*, 59:99–134, 1995.
- [8] Klemperer, P.D. and Meyer, M.A., Supply function equilibria in oligopoly under uncertainty. *Econometrica*, 57(6):1243–1277, 1989.
- [9] Newbery, D.M., Power markets and market power. *The Energy Journal*, 16(3):39–66, 1995.
- [10] M. Yang and E. G. Read. A dual dynamic programming approach to reservoir management with correlated inflows. In *Proceedings of the 26th Conference of the ORSNZ*, pages 21–26. Operations Research Society of New Zealand, 1990.