Dual OR: a Partial Paradigm for the Future?

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Abstract

Over the past few years many authors have documented the demise of some traditional forms of OR practice. But at the same time, significant success has been reported in other areas. It is suggested that this pattern of failure and success reflects the degree to which OR has proved adaptable to a shift away from a reliance on top-down planning, toward reliance on pricing mechanisms; that is from a "primal" to a "dual" orientation, both in business and in Government. If so, this suggests significant potential in the adoption of a Dual OR paradigm which emphasises the use of OR to create and support "market" structures within which independent decision-makers can operate. Conversely, though, the OR profession must ask whether its current tool-kit is really adequate to the job ahead.

1 Introduction

The concept of duality has been central to mathematical programming, and hence to OR since its inception, and its economic interpretation is well known. Indeed for many years there was a strong interaction between Economics and Operations Research, with unified textbooks such as Baumol's "Economic Theory and Operations Analysis" appearing in successive editions [1]. Operations Research was even housed in an Economics Department! In recent years, though, the two disciplines seem to have largely parted company, with Economics becoming very much the dominant paradigm of national and international discourse over the last decade, while Operations Research seems to have developed a pre-occupation with defining its own role and future. It may be suggested that these two developments are not entirely coincidental. Specifically, modern economics has created an environment in which traditional OR has found it difficult to thrive. Conversely, it is argued that the failure of OR to appreciate and build on areas of common understanding between OR and economics has been detrimental to the success of OR.

This has much to do with the type of economics which is now being practised. There was a time when mainstream economists and OR people could sit down together and seriously discuss the "optimisation of national welfare", on the assumption that a national welfare function existed, and could be at least approximately specified. An elegant price driven decomposition could be developed, starting from that common base, and the

relative advantages of "implementing" that solution via centralised planning, or a "free" market could be seriously debated. This is no longer the case, for two reasons. First, the economics currently in vogue denies the philosophical possibility of constructing a social welfare function to be optimised. Second, experience suggests that centrally planned economies have proved woefully inadequate in meeting the challenges of a rapidly changing world. Thus modern economics puts a very strong emphasis on creating market structures which give individuals and firms the freedom to maximise their own welfare while, hopefully, limiting any party's "market power" to exploit others. Thus the central focus is on the use of prices to provide both information and incentives to all parties, and on the institutional framework which makes those "price signals" effective by making contracts enforceable, for example.

Critics may carp at various aspects of this paradigm, but there is no denying its current success in terms of generating economic growth (even if national welfare can not be defined!), making it unlikely to be repudiated in the near future. So where does that leave OR, with its traditional emphasis on top-down optimisation? There are certain types of organisation and activity for which this orientation is still appropriate, and where advances in OR technique and IT capabilities have combined to produce continued success. But there are other areas in which OR has struggled, and often lost its traditional role.

This paper argues that the OR community could gain significantly from a belated attempt to re-creating at least a partial synthesis with economics, and re-orienting the OR profession toward what might be referred to as "Dual OR", or "OR as if economics mattered". Indeed, there have already been enough successful developments along these lines to suggest that the concept provides a promising paradigm for the future. It is only a partial paradigm, though, because it does not directly address the whole range of opportunities and threats facing the OR community. The concept of Dual OR was first suggested by Read [23,24] in the context of electricity sector developments, and briefly outlined in a more general context by Daellenbach and Read [8]. This paper, which draws extensively on the latter, re-iterates the concept, and tentatively explores its relationship with other trends and paradigms currently influencing the OR profession. Thus it should be seen more as a discussion starter than a pronouncement on the future of OR.

2 Review

Daellenbach and Read, writing in response to an article by Fildes and Ranyard [13] surveying the rather troubled state of OR in the UK, suggested that the state of OR is closely related to fundamental changes in the economic climate. Particular attention was drawn to a new management culture emphasising leadership, innovation, and change management, in the context of a dramatic shortening of the product, project, and organisational life cycles. Accordingly, it seemed appropriate to ask how OR and its practitioners might need to adapt to this new economic environment and its associated management style and culture. Two critical implications were suggested.

First, the development cycle for OR models must be short enough to provide a timely response to demands by management for advice and help in decision making. The role of spreadsheet based packages, and of modelling languages like AMPL, GAMS, or AIMMS, was acknowledged, but the challenge remains critical.

Here we concentrate on the second implication, which relates to the issue of systems-wide modelling and what was termed "primal versus dual control" over decision making. It was noted that much modern managerial thinking concentrates on the role of "prices" or more generally "incentive mechanisms" to achieve the optimal allocation of scarce resources. By way of contrast, even though the idea of decomposition via a price mechanism for resource allocation is nothing new to OR, the overwhelming emphasis has been on "solving the (primal) optimisation problem", with the output from any decomposition model still typically being implemented via system-wide control from the top. Hence the paradox that, while the "real world" has increasingly adopted the concepts of "duality" and "price decomposition" so effectively pioneered in the OR community, mainstream OR literature seems to have concentrated on ever larger and more complex approaches to primal optimisation. Consequently, a Dual OR approach was suggested, in which the decomposition approach was accepted as a philosophy of organisational structure, leading to decentralised decision making, in which OR would assume a new role where:

- *Rather than "solving a primal problem" to determine what lower levels in the hierarchy should do, and then ordering them to do it,*

- *A structure is designed so as to give incentives (loosely "prices") for all parties to act in a way which furthers organisational or societal objectives.*

It was noted that the first mode relies on the existence of a large organisational structure with a central command and control mechanism. This leads to building system-wide models that contain all significant interactions between various parts of the organisation. They aim to optimise a system-wide performance measure. But, it was argued, the second mode would focus on freeing and stimulating individual initiative by delegating substantive decision making to individual operating units. This shift in thinking results from a perception that the very existence of a centralised, hierarchical command and control structure, by virtue of its inertia and impact on deadening incentives for innovation in the field, can easily outweigh any gains which might be made from a global optimisation within such a structure. This perception, it was suggested, is greatly reinforced now that the environment, structure, and technical characteristics of the "system" are in a constant state of flux, so that centralised decision-makers are increasingly less well informed than the aggregate of the individuals involved. The authors then went on to consider the threats and opportunities which these developments imply for the OR community, and to reflect on the implications for OR practice, and for the training of OR professionals. Briefly, a current and future need was identified for three quite different types of OR practitioner:

- First, a relatively small number of "conceptual advisers", trained in both OR and economics, and at least comfortable with the key concepts of modern managerial culture would design, assess and interpret structural innovations, often in the form of internal or external "markets". They would need to have "commercial credibility" and project management skills, and be able to explain and "sell" OR concepts to a diverse and wide range of audiences,

- Second, would be small specialised groups devoting their mathematical and computer skills to the development of OR software.

- Finally, there would be a much larger group of flexible, innovative, and commercially competent consultants, operating within the environments established, at least in part, by higher level OR activity, using commercial OR products.

This paper reconsiders aspects of this vision, in relation to other trends in the OR world. But first a reality check. Is there any real-world evidence to suggest that this kind of development could be effective, or worthwhile?

3 Real world experience

The concepts advanced here are an outgrowth of local experience in the electricity sector, as documented by Read [23], who lists six areas of increased OR activity as a result of deregulation in the New Zealand electricity sector. The central feature of this sector is an LP model "clearing the market" to determine half-hourly dispatch on the basis of offers and bids from generators and reatiler/customers, and to set half-hourly prices for electricity at each supply point in the country. This represents the quintessential "dual" OR application, in which the goal of the centralised model is not to determine what each unit should do on the basis of, supposedly superior, centralised information, but to provide an environment which allows a wide variety of participants to coordinate their activities, determining prices to provide a consistent basis for trading between them, and conveying the information and incentives required to improve their operation. Read and Chattopadhyay [25] reflect further on experience with such models. But, clearly, the use of a classical LP model for coordination gives considerable prominence to the role of OR, and means that market participants must understand such techniques.

This centralised OR model is complemented by a wide range of traditional OR applications, such as cost minimisation and market simulation, which are now being done on a smaller scale, but by many more parties. There is also an increased emphasis on "dual" application areas, such as price optimisation and using the insights derived from OR models to study issues of organisational and incentive structures. The net result is a significant increase in OR activity in the sector. It might be argued that much of that activity is of "lower quality", but that is not really the point. Certainly, simpler models are now used for smaller components of the systems. In most cases this is more than offset by the fact that local management pay far greater attention to getting the data right, and getting the most out of their particular sub-system. Although devolution of decision making has weakened the position of formerly central (and often powerful) OR modelling groups, overall there are now more OR people employed, there is a much wider variety of OR techniques used, and there is an increased rate of innovation.

This experience suggests that it is possible to adapt successfully to the new environment by re-interpreting our traditional techniques to provide a "dual" approach to OR, which emphasises two things. First, centralised modelling can play a critical role in analysing, establishing, and maintaining the environment in which the decision-making units will operate. Second, this must be complemented with the provision of tools which help relatively small decision-making units respond in a timely and effective manner. Most importantly, though, this experience has not been unique to New Zealand. Although it gives less prominence to mainstream OR technology, a similar regime has been operating for some time in the UK, where it has also been a focus for significant OR activity (see Bunn [4] and Littlechild [19]). In Australia, an LP based market has also been implemented for the electricity industry, and another for the Victorian state gas

sector. Very similar approaches have been adopted in major US electricity markets, such as California, New York, and New England, and are now being pursued in Ontario, South East Asia, for example.

Nor is this situation unique to the energy sector. As noted earlier, many organisations, small and large, now think in terms of "optimising" the design of, and response to, pricing and incentive schemes, both to interact with markets and to co-ordinate their own internal activities. "Yield management" has recently leapt into prominence as a critical success factor or airlines, and a critical focus of OR activity in that sector (Cook [6], Boyd [3]). This type of price optimisation, which parallels work on optimising bidding strategy in the electricity sector, is another quintessential expression of the Dual OR paradigm. In fact a quick survey of recent issues of mainstream OR journals reveals a significant component of articles on topics lying on the interface of OR and economics. Examples include Dada and White [7], Dial [9,10], Cachon and Zipkin [5], Federgruen and Heching [11], Feng and Xiao [12], Jain et al [14], Jing-Yuan and Smeers [15], Keeney [16], Lee [1999], Li and Kouvelis [18], Masuda and Whang [21], Van Mieghem [29], Petruzzii and Dada [22], Smith and McCardle [28], Vilcassim et al [30]. Thus, even if Dual OR has not been recognised as a distinctive stream within the OR community, there is already a significant interest in such topics, and considerable success being experienced in some areas.

Finally, Daellenbach and Read also note the way in which large-scale structural change of the type occurring in the electricity sector has been paralleled at quite the opposite end of the spectrum, on the factory floor, where much activity now focuses on creating an environment in which individuals and small groups are encouraged to perform and innovate, rather than being treated as variables, or perhaps constraints, in a "primal model". Even if no formal optimisation is involved at any level in the organisation, this too may be seen as an important, and very widespread, expression of the trend identified here as Dual OR.

4 Conflicting trends

So, having perhaps established some plausibility for the proposition that Dual OR provides a unifying theme for some current trends in OR practice, and hence a possible paradigm for future OR development, how does this paradigm fit with other paradigms current in the OR world, and why is it only a partial paradigm? We have already tried to identify what makes this approach different from the traditional OR "paradigm". But the dividing line between these two is not always clear, with many projects, past and present, exhibiting aspects of both. Nor should we be surprised to hear that "traditional OR is alive and well at XYZ corporation". Long may it be so. We have acknowledged, too, the validity and vitality, and indeed the necessity, for what might be called "OR on the fly"; applying off the shelf tools to deal with problems as they emerge in an ad hoc fashion. Conversely, it should be clear that that we do not subscribe to the "Death of OR" school of thought, in its various guises. Certainly some parts of OR will die, but others will expand, and the discipline still has much to contribute. But it is probably worth considering how Dual OR relates to three other paradigms with which it may seem to be in conflict

Strategic OR

First, Bell [2] has recently advanced the concept of "strategic OR/MS" as a future direction for OR activity, and local experience would heavily underline the critical importance of "gaining the high ground" in an organisation, thereby establishing an environment sympathetic to lower level OR activity. He rightly notes that corporations who put a high value on such activity are likely to want to keep it in-house, which contrasts somewhat with Daellenbach and Read's comments on the growth of consultancy, although strategic consultancy is big business too. But the general thrust of development in strategic OR must be seen as very much in line with the Dual OR paradigm advanced here. Modern business strategy is as much about pricing as it is about production, and it is very much about corporate restructuring and incentivisation for productivity and innovation. On the other hand, "dual" OR activity encompasses tactical as well as "strategic" activities. Thus, far from being in conflict, these two concepts may be seen as complementary descriptions of closely related phenomena.

Enterprise Engineering

This term is used here to refer to the integrated top down "engineering" of business structures and processes to achieve a design goal. Thus it includes such movements as Business Process Re-engineering (BPR) and Enterprise Resource Planning (ERP). In part, this broad approach shares goals with Dual OR, and may be seen as an effective means of achieving those goals. But, increasingly, the focus and end result of such exercises may be encapsulated in the formation of an integrated Management Information System (MIS) which, while it may facilitate interaction between independent units, may also be used as an instrument to effect centralised control on an unprecedented scale. Indeed such developments are already common in the electricity sector, where automation and "de-manning" of all power stations in a river system, for example, now enables real time control direct from a single control centre.

This kind of development offers tremendous opportunities and challenges for OR, as recognised by Robinson and Dilts [27] who rightly conclude that "..the OR profession (must) take notice of this important change and rapidly show how OR can both benefit and be a benefit to ERP." On the one hand there is no need to go and "find the data" any more.. the data is there in super-abundance. On the other hand OR practitioners must learn to interface their models to the MIS, rather than expect it to be modified to their requirements, and face the threat that the existence of a good MIS within an organisation can create the appearance that all issues may be addressed by simplistic manipulations of the vast stream of data focussed on the CEO's desktop, leaving no apparent need for the kind of analyses which OR can offer. This, in turn, emphasises the importance of a strategic approach, which ensures that OR is represented at the highest organisational level, and so shapes MIS development.

But how does this movement relate to the Dual OR concept? It may be suggested that the Dual OR concept relates more to the way in which sectors are structured, and to the way in which units within that structure relate to one another, whereas the "enterprise engineering" philosophy, in its various forms, relates to the efficient internal organisation and management of the individual units. Thus the Dual OR paradigm envisages OR occurring in the context of an economy typified by many interacting organisations, each having shed its non-core activities to focus on the its core business.

Enterprise engineering is very much about focussing on core business, to the exclusion of all else, and is thus complementary to, rather than in conflict with, the Dual OR concept. It will be interesting to see how the balance develops between the ever expanding technological capability to facilitate centralised control over larger organisations, and the equally rapid expansion of the technological capability to support decentralised interaction between independent organisations. Both will be critically important in shaping the future of OR.

Systems Thinking

Finally, Daellenbach and Read note that Dual OR concept represents a rather drastic change from the ethos of the operations researchers of the sixties and seventies, who liked to see themselves as systems thinkers aiming at global optimisation. Nor does it seem particularly consistent with the extensive literature on systems thinking, and its emphasis on holistic decision-making. Here there is, I believe, a genuine and distinct difference in approach, and one driven by the simple observation that, at least in this author's experience, there is very little interest in systems thinking from "the other side"… that is from real world decision-makers. I suggest that this lack of interest stems from three factors:

- First, the systems literature tends to recommend processes which involve more people, for more time, than businesses currently feel they can afford.

- Second, even when management teams are prepared to devote that kind of time, typically for strategic planning it is not obvious (to them at least) that methodologies developed by "OR" specialists offer any distinct advantage over those developed by specialists in, say, strategic planning.

- Third, and most importantly, no matter what their personal values may be, professional managers typically consider it inappropriate to consider the (dis)benefits which any proposal might bring to other parties, except inasmuch as this provides some leverage, or hindrance, with respect to maximising benefits for their own organisation… which they see as their legal, and arguably moral, responsibility.

Members of the OR community may be unhappy with this situation, but that will not change it. It may be that, in the year 2100, our children's children will look back on this era with pity or disgust. It may be that there will be no-one left to look back in 2100, without a major change toward a more holistic ethic of decision-making. It may be, in other words, that the prophets of systems thinking will be vindicated when it becomes the dominant paradigm of the next century. But, it seems obvious that systems thinking is simply not the dominant decision-making paradigm of the nineties. Nor do I think it likely to gain much ground in the "noughties" to come. The problem is that, not only is their limited public, or coporate, willingness to accept such a paradigm, but the institutions required to establish, let alone implement, consensus solutions on the major problems facing our society simply do not exist. Nor, given the increasingly global nature of the problems, can effective institutions be established at anything less than a global level. The rapid development of such institutions seems unlikely, and it is noteworthy that the current thrust of such global institutions as do exist is, on balance,

much more inclined to promote the internationalisation of the market paradigm than the kind of holistic approach which systems theorists might advocate. Thus the OR community must seriously consider how it should conduct itself in such an environment.
In breaking away from the rigidity of "hard OR", the systems movement often seems to move all the way to the opposite extreme by focussing on the "messiness" or "wickedness" of problem situations involving multiple conflicting parties, value systems and perceptions of reality. Such problems are real, pervasive and critically important, but great effort will be required to yield any measure of success, or even to convince decision-makers that they are worth tackling seriously. Conversely, it has been suggested here that the interface between management science and managerial economics represents a relatively fertile field for development, and one for which adaptations of traditional OR methodology seem more likely to be successful. There would also seem to be room for the application of systems thinking within the Dual OR environment described here (or arguably vice versa!). This is particularly so when designing market "systems", where the application of systems thinking carries the promise of providing at least partial solutions to some "messy" problems by developing market structures to deal with critical constraints for example. Witness the establishment of markets to trade pollution rights, or fishing quotas. At this point in time, though, the credibility of "systems thinking" among decision-makers seems dependent on overcoming the perception that the benefits it offers will be outweighed by the deleterious effects of imposing a centralised decision-making regime capable of undertaking and implementing the outcome of holistic planning.

5 Conclusion

It has been argued that increasing emphasis on a dual approach to OR, combining aspects of traditional OR with economics, may provide a partial paradigm for OR practice in the present, and immediate future, in which economics provides the dominant paradigm for decision-making. Of course these ideas are not entirely new. Antecedents may be found in the use of policy models to simulate, and set parameters for, the national economic environment (see Manne et al [20] for example), and in the "shadow" or "transfer" price mechanisms sometimes used to coordinate decision-making in large corporations or centrally planned economies, as in Read et al [26], for example. Still, the use of market mechanisms is now more pervasive at all levels of the economy, and on a virtually global basis. Hence the importance of OR re-discovering some degree of commonality with economics, and striving to contribute positively in the current environment. Fortunately, the evidence suggests that this kind of activity is indeed increasing in some areas, although there is a long way to go before Dual OR could really be said to exist as a cohesive and comprehensive paradigm for OR activity. More importantly, though the OR community needs to establish and maintain its reputation as a distinct and positive contributor with respect to such issues. There are three basic requirements:

- First, OR will only be successful if it understands, respects, and interacts constructively with the prevailing climate and managerial culture.

- Second, OR practitioners must be able to deliver something distinctively different from economists, or from general management consultants.

- Third, OR practitioners must gain and retain the respect of influential advisors in those other disciplines.

It may be suggested that this third, and arguably most important, aspect flows from the other two, in that OR practitioners will only be respected, and hence consulted, if they respect other disciplines, and make a distinctive, valuable and timely contribution. The ability to deliver significant and timely results continues to rest very heavily on the tools available to, and training of, the OR practitioner. OR has seen tremendous computational, algorithmic, and implementational advances in recent years. But we may still question the adequacy of the OR toolkit, and training, to the task which now faces us. In particular, Daellenbach and Read suggest that our traditional toolkit has proved surprisingly poor at coping with the central problem faced by modern management, namely the maximisation of some (admittedly limited) kind of human potential in a world of increasingly turbulent structural and technological change. Our traditional clients may call on us to assist them in this new challenge, but many of us will be quick to tell them that we cannot analyse (and therefore disapprove of!) the new structures they wish to "optimise", because they involve mathematical complexities, such as non-linearities, non-convexities or discrete alternatives, which invalidate our finely tuned optimisation algorithms. Worse yet, the key variable they are concerned with maximising, i.e., the impact of a structure in terms of empowering and incentivising their people to innovate, simply does not appear in our models. There is clearly some way to go if the Dual OR concept is to achieve its apparent potential.

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