# A Model for Household Electricity Demand-side Optimisation

Erling Pettersen
Norwegian University of Science and Technology
email: erlingp@iot.ntnu.no

**Abstract**

This paper presents a Stackelberg game model of strategic interaction between an electricity retailer and an end-user in an electricity pool market. The retailer offers a vector of 24 hourly prices to the end user who lives in an intelligent house. The intelligent house minimises the electricity costs based on the offered prices taking into account the residents' habits. The consumption pattern chosen by the consumer must be purchased by the retailer in the risky wholesale market. We assume that the risk averse end-user is unwilling to take the risk in the wholesale market while the risk neutral retailer seeks to maximise expected profit. Thus the price profile offered includes a risk premium. To model the house's cost minimization we have chosen to view the house as an energy storage, where space and water heating are carried out at the lowest possible cost, an approach which results in the house solving a linear program. This is convenient due to the highly non-linear and non-convex nature of the mathematical program with equilibrium constraints (MPEC) which constitutes the overall problem.

## 1 Introduction

In deregulated electricity markets the severe competition in the retail industry has triggered the development of sophisticated models for managing the risk in the wholesale market. Demand-side management efforts have mainly focused on the industrial sector where the customers have high individual demands. There is, however considerable potential for demand-side management programs in the residential sector. Asano et al [1] measure the effect of incentive payments on residential time-of-day (TOD) electricity demand in the Kyushu region in Southern Japan. Results based on an econometric model suggest that households tend to modestly shift their electricity consumption from peak to off-peak when offered incentive payments for load shifting. Filippini [2] considers the household on a micro level and expresses the household budget shares of peak and off peak consumption as a function of electricity prices, real electricity expenditures and household characteristics. The results indicate that the demand for peak and off-peak electricity is elastic.

In [1] and [2] the households are offered two tariffs: an off-peak price and a peak price. The present study however considers a household whose electricity consumption is metered in real time. With real time metering the price signals in the wholesale market may be embedded in the price profile offered to the households by the retailer. In this paper we develop a plausible model of domestic demand-side response to the price signals in the market. To represent the flexibility in some of the house's electricity consuming processes we view the house as an energy storage. To cut energy costs the consumer may choose to heat space and water some hours before the heat is needed. In our model, this preheating is equivalent to filling up the energy storage. Energy will flow out of the storage due to the difference between the inside and outside temperature and to the consumption of hot water.

The load shifting could be done in different ways. One simple solution would be to connect timer devices to space heaters and water heaters. A more sophisticated solution would be to assume that the consumer receives a price vector from the retailer and, based on this vector and the end-user's consumption preferences, the cost of electricity consumption is minimised. At least in Scandinavia it is becoming increasingly common for end-users to install advanced technological solutions capable of automatically carrying out the cost minimisation. These solutions are also designed to turn the heating devices on and off, implying that the residents' inconvenience of regulating the load levels, and therefore an implicit transaction cost, is removed. Within this framework the retailers will be able to provide the end-users with proper price signals and the end-users will be able to react to these price signals in an efficient manner. As it is believed that the market for such solutions, sometimes referred to as "smart houses," will increase over the next years, we choose this advanced approach to load shifting. We refer to the web page [4] for more information on smart house solutions.

Using this model of demand-side response we look at demand-side management from a domestic perspective. We consider the contracting decision made by a retailer which sells electricity to a real time metered end-user in a competitive market environment. The retailer must decide on a price vector to pass on to the end-user who in her turn, based on the offered price vector, minimises the daily cost of electricity consumption. This interaction is modelled and implemented as a Stackelberg game model.

In Section 2 we present the model of domestic demand-side response. For extra motivation we have included some reflection on the recent energy problems in New Zealand in this section. Section 3 describes a game model of demand side management and Section 4 concludes this paper and presents some ideas for future research.

## 2 A model of domestic demand-side response

End-users utilise electricity for many different processes: refrigeration, space heating, water heating, cooking and laundry to mention a few. Some of these processes are more flexible than others when it comes to changing the time the process is accomplished. For example we may think of the laundry as a process that can be moved to a time of the day when the electricity prices are low while lights must be on when the residents are in the room no matter how high the price is. Other important examples of flexible processes are space heating and water heating. After the living room is heated up, it will take some time before it gets uncomfortably cold and water that is heated may still be warm enough for a shower after some hours.

In this model we focus on the flexibility of water heating and space heating and combine these two processes by thinking of the house as an energy storage. Energy flows out of the system because the energy level inside is higher than that on the outside (we assume that it is colder outside than inside). The higher the difference between the energy level inside the house and outside the faster the energy will flow out. When space heaters and/or the water heater are turned on, energy comes into the storage.

Mathematically we model this as

$$M\frac{dE(t)}{dt} = Q(t) - \alpha(E(t) - E_{out}(t)) \tag{1}$$

where
$E(t)$ is the energy level at time $t$
$Q(t)$ is the power that is turned on to provide energy to the storage in time $t$
$E_{out}(t)$ is the energy level outside the house
$\alpha$ and $M$ are positive parameters.

For simplicity we assume that the temperature outside is constant throughout the day. This means that we may think of $E_{out}$ as the reference level of energy and without loss of generality assume that $E_{out}(t) = 0$. It should be emphasised that this assumption is made for simplicity. In reality the outside temperature would effect the demand because the end users would adjust load to temperature and load tends to have an influence on the price. A more realistic approach, which may be considered at a later stage, would be to model $E_{out}$ as a random parameter.

Discretising (1) and solving for $E_t$ gives

$$E\left(t\right) = \frac{\Delta t}{M}Q\left(t\right) + \left(1 - \frac{\alpha\Delta t}{M}\right)E\left(t-1\right). \tag{2}$$

where $\Delta t$ is the length of one time step.

When water is consumed the system will experience a direct outflow of energy. This is incorporated in our model as follows:

$$E_t = \frac{\Delta t}{M}Q\left(t\right) + \left(1 - \frac{\alpha\Delta t}{M}\right)E\left(t-1\right) - E_W\left(t\right) \tag{3}$$

where $E_W\left(t\right)$ is the amount of energy lost by the system due to water consumption in time period $t$.

The residents have some comfort requirements that serve as constraints in our model. These constraints are modelled by specifying required energy levels for each hour of the day. The house must maintain the energy storage so this requirement

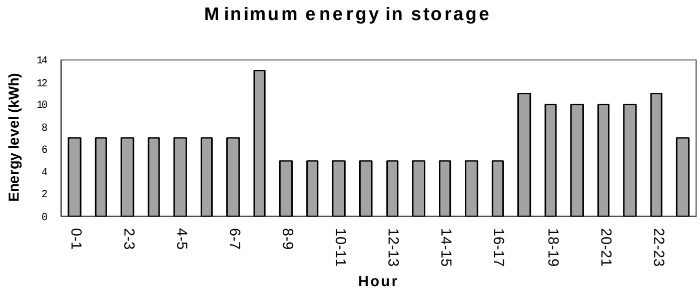


Figure 1: The energy level required by the household. The residents need the house to be warm and comfortable when they are at home, but when asleep at night the house may be colder. The energy required is highest in the hours in which we assume that hot water is consumed.

can be met at all times. The required energy levels in this model are shown in Figure 1. We assume that the requirements do not change in the short run.

We let $P_i$ be the electricity price in price period $i$ and $E_{req}(t)$ the required energy level at time $t$. The optimisation problem for the house then becomes an LP over $I$ trading periods:

$$\min C(P,Q) = \sum_i P_i \left( \sum_{t \in i} Q_t \Delta t \right) \quad ,i = 1,2,...,I \tag{4}$$

subject to

$$\begin{aligned} ME(t) + (\alpha\Delta t - M)E(t-1) - \Delta t Q(t) &= -ME_W(t) &(5)\\ E(t) &\geq E_{req}(t) &(6)\\ E(t) &\leq E_{\max}(t) &(7)\\ Q(t) &\leq Q_{\max}(t) &(8)\\ Q(t), E(t) &\geq 0 &(9) \end{aligned}$$

where the constraint (5) is Equation (3) rearranged to move the variables to the left hand side. $C$ is the cost of energy consumption and with $t \in i$ we mean the time periods $t$ that belong to the price period $i$. The presented model, though not perfectly realistic, is meant to serve as a plausible representation of domestic demand-side response.

To illustrate how this model performs we will now have a look at the behaviour of a real time metered household in the New Zealand electricity market. In New Zealand the abnormal weather conditions this winter have caused the hydro reservoir levels to run very low. The shortage of supply, combined with the high demand resulting from a cold winter, has lead to wholesale electricity prices that are very high by New Zealand standards. In addition the prices have shown considerable variation over the day with relatively high prices early in the morning and extremely high prices at the start of business hours and in the evening. To illustrate, Figure 2 shows the wholesale prices at the node "Haywards" on the 24th July this year plotted together with the prices at the same node and the same date in 2001.

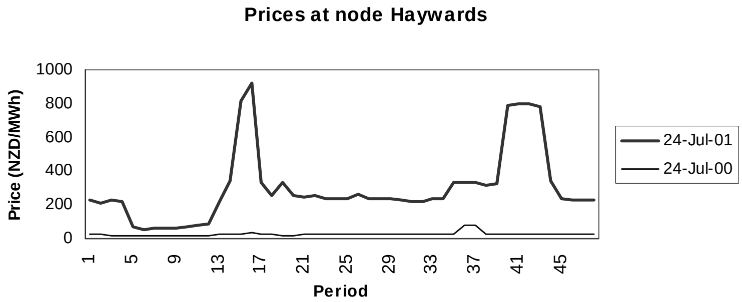


Figure 2: Prices at node Haywards on 24-Jul-01 (thick line) and on 24-Jul-00 (thin line).

If we assume that the end user's electricity consumption is metered in real time it would not be unreasonable to assume that the prices in Figure 2 are passed on to the end user. Making this assumption we offered the price profile of the 24 July 2001 to the house. For comparison we offered the prices on the 24 July 2000. We then offered a flat rate of $297.21 NZD/MWh$, which was the average price the 24 July 2001. When offered a flat price, the cost minimisation is equivalent to minimising the total energy consumption. We also slightly revised the model to minimise the total energy consumption and calculated the cost of this strategy when the market price is offered for each period.

The results are presented in Table 1. When consumers pay an average price (as they tend to do without real time metering) they seek to minimise total daily consumption. This gives a consumption of $67.7kWh$. When charged at the average price this amounts to \$20.11, but \$25.33 at the market price. This cost can be reduced by real time metering whereby the household pays \$17.81 but consumes more. (This depends on the price profile, c. f. the cost for 24 July 2000 is \$1.68.) Thus the household will make significant savings if it utilises the price profile and uses energy when the price is low compared with if it just minimises the overall consumption. The household is also better off when given the opportunity to utilise the price profile than when it is offered the flat rate. Figure 3 shows how the price profile is utilised by the end user. We clearly see how the cost minimising end user chooses to warm up the house when the prices are low, while the consumption minimising end user would warm up the house just in time to satisfy the comfort constraints. The consumption profile when minimising consumption is equal to that when offered a flat price.

From a society perspective it may seem disadvantageous if the end users behaved this way since the energy consumption would rise. If we consider the situation in New Zealand last winter, the problems were caused by low water levels in the hydro reservoirs. Hence, the main focus should be to save hydro power. The New Zealand electricity system is a combined hydro-thermal system and as the situation was in the winter, it would be beneficial to society to trade hydro production for thermal production. In the low price hours there was some excess thermal capacity, which means that the end user studied above actually has traded hydro power for thermal

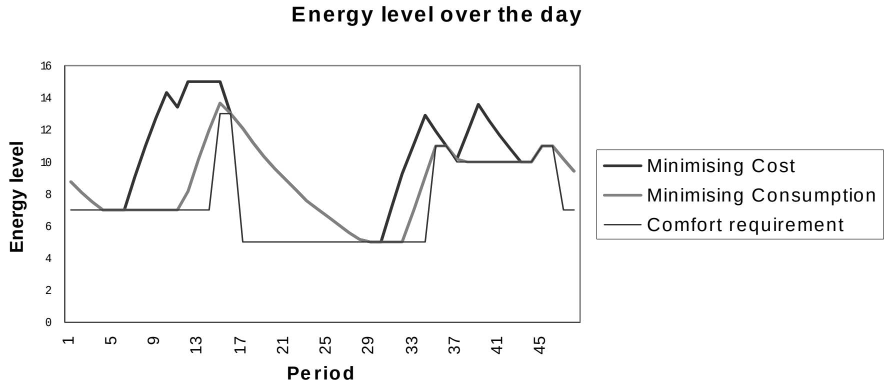


Figure 3: The picture shows how the consumption profile for the end user when minimising the cost is significantly different from the profile when minimising consumption.

| Scheme | Cost (NZD) | Consumption (kWh) |
|---|---|---|
| Flat rate, the average price of 24 Jul 2001 | 20.11 | 67.7 |
| Minimise consumption. Market prices | 25.33 | 67.7 |
| Market prices 24 Jul 2001 | 17.81 | 77.3 |
| Market prices 24 Jul 2000 | 1.68 | 72.8 |

Table 1: The end user minimises the cost of electricity consumption based on three different pricing schemes. The last row shows the results of minimising energy consumption when offered market prices.

power and thereby saved water. When more and more end users behave like this, we will gradually reach a point where the excess thermal capacity is consumed and hydro power would be traded for more hydro power. This situation would however be reflected in the market prices, which again would make the end users behave differently.

In normal circumstances hydro power is produced at a lower marginal cost than thermal power. Thus, if end users are able to react to price signals as efficiently as described, more hydro power and less thermal power would be consumed. This represents an improvement in overall welfare even though more energy is consumed, since the prices are (at least in theory) computed to maximise society's benefit.

# 3 A game model of demand side management

The existence of real time metered end users with the ability to respond to price signals as described in the previous section would introduce some new possibilities and challenges for the retailer. The possibilities may include a variety of new products which could be designed to meet the desires and preferences of groups of customers or even individual customers. An important challenge would be for the retailer to anticipate the end user's reaction to whatever product she is offered.

The model presented in this section considers a retailer operating in a competitive environment and one end user purchasing electricity from the retailer. The purpose of the model is to study the strategic interaction between the retailer and the end user. First, the retailer decides a price profile to offer the end user and next the end user makes a consumption decision. Hence, the presented game is a Stackelberg game with the retailer as leader and the end user as follower.

## 3.1 Assumptions

In this subsection we explain some important assumptions underlying the study. First we present our assumptions regarding the electricity prices. Next a few necessary assumptions are made for the players.

### 3.1.1 Electricity prices

We assume that the retailer operates in a deregulated electricity market. A common feature of deregulated electricity markets is that there will be a forward price and a final price. The forward price will reflect the expectation of the final price some time in advance while the final price reflects the marginal cost of delivering electricity to a certain area.

In our study we assume that there are 8 equally likely scenarios for the final price. The forward price is the expected value of these 8 scenarios.

### 3.1.2 Assumptions on the players

We assume that the the demand is derived from disutility minimising behavior by the end user. The disutility from electricity expenses is expressed as

$$D(C(P,Q)) = e^{\gamma C(P,Q)} \tag{10}$$

where $D$ is the disutility, $\gamma$ is a positive parameter and $C(P,Q)$ is the cost of electricity consumption as a function of price and consumption as in Equation 4. The disutility function is convex for all $C$, implying that the end user is risk averse. When the end user is offered a deterministic price, minimising disutility is equivalent to minimising cost.

The retailer is assumed to be risk neutral and therefore its goal is to maximise expected profit. We also assume that the retailer has perfect knowledge of the parameters in the end user's disutility minimisation problem.

## 3.2 A Stackelberg game model

The retailer's task is to decide a price profile $p$ to offer the end user. Based on $p$ the end user decides a consumption profile $Q\left(t,p\right)$. The retailer must purchase $Q\left(t,p\right)$ in the wholesale market at a random price $\widetilde{p}$. Since the retailer is risk neutral, its objective is to maximise expected profit and hence the retailer's objective function becomes

$$\max \sum_{i} \left(p_i - \overline{p}_i\right) \left( \sum_{t \in i} Q\left(t,p\right) \Delta t \right) \tag{11}$$

where $p_i$ is the price offered by the retailer in time period $i$ and $\overline{p}_i$ is the forward price in period $i$.

To account for competition we assume that the end user may choose to take the risk in the wholesale market instead of accepting the price profile offered by the retailer. Thus to ensure that the end user prefers the retailer's offer to the wholesale market, the retailer should offer a price profile which satisfies the following constraint

$$D(p, Q(\cdot, p)) \leq E\left[D\left(\widetilde{p}, Q\left(\cdot, \widetilde{p}\right)\right)\right] = k. \tag{12}$$

In Equation 12 the left hand side represents the end user's disutility from accepting the price profile offered by the retailer. The right hand side is a constant expressing the expected disutility from purchasing power directly from the wholesale market.

Furthermore the retailer knows that the end user, whatever price profile she is offered, will choose the variables $Q(t, p)$ so as to minimise costs. For the solution to be optimal for the retailer we need a constraint which ensures that the solution is optimal also for the end user. Hence we need a constraint to ensure that we have found an equilibrium solution to the game and therefore we need to add the following *equilibrium constraint.*

$$Q \in \mathcal{S}(p) \tag{13}$$

where $Q(t, p) \in Q$ for all $t$ and $\mathcal{S}(p)$ is the set of optimal solutions for the end user dependent on $p$. The constraint (13) tells us that $Q$ has to be the set of optimal loads for the house for any price profile $p$ offered by the retailer. We have pointed out that given a deterministic price profile the disutility minimisation and cost minimisation are equivalent. Therefore in this case the constraint (13) ensures that is an optimal solution to the LP (4) - (9). The program (11), (12) and (13) is a Mathematical Program with Equilibrium Constraints (MPEC). We refer to Luo et al [3] for a treatment of MPEC problems.

## 3.3 Optimisation results

According to Tin-Loi and Que [5] there are three features of MPEC models that make them difficult to solve. Firstly, the equilibrium constraint (13) is modelled by writing down the complementarity slackness conditions of the LP (4) - (9). However the complementarity constraints are disjunctive, making the feasible region a union of infinitely many closed sets. Secondly, the feasible region of the MPEC may be non-convex even if all functions defining it are "nice." Thirdly, the feasible region may not be connected. These three difficulties are often expected to show up as a severe numerical instability.

In our search for an optimal solution we may use the fact that we already know one local optimum. Since the end user is risk averse, the retailer can make a positive expected profit by providing insurance to the end user. A trivial way of doing this would be to offer the expected wholesale price plus a risk premium in the form of a mark-up $\xi$ on the final price. This policy is common in today's deregulated electricity markets. Since the retailer has complete information on how the end user will respond to any price profile offered, the retailer may maximise its expected profit by choosing the mark-up $\xi$ which exactly makes the end user indifferent between the retailer's offer and the risky wholesale market. In this case the optimal $\xi = 0.312$, giving an expected profit of $22.41NOK$.

The numerical instability was indeed a problem when solving present model. When the retailer was given the opportunity to offer 24 different prices, that is one price for each hour, even small changes in starting solutions gave totally different results. Indeed, the price profiles suggested by the model did not look like anything a retailer would seriously offer to an end user, even if the expected profit of $22.83NOK$ was slightly higher than the solution from the constant mark-up approach above.

To reduce the number of variables we divided the day into four pricing periods. This did give price profiles that looked more like something a retailer would be willing to pass on to an end user, but the solutions are still very unstable. Table 2 shows one of these solutions while Figure 4 shows the energy level over the day in the constant mark-up approach compared to the solution from the Stackelberg game with four pricing periods. The price difference between the first two periods is rather small in this solution and therefore the house starts heating the house only slightly earlier than necessary. However the price difference between the last two periods is more significant and the heating is started a bit earlier than necessary to save cost. The retailer may charge a relatively high price in the last period because the end user must consume electricity to maintain the required comfort level. The retailer knows that the end user must consume and utilises this by charging a high price.

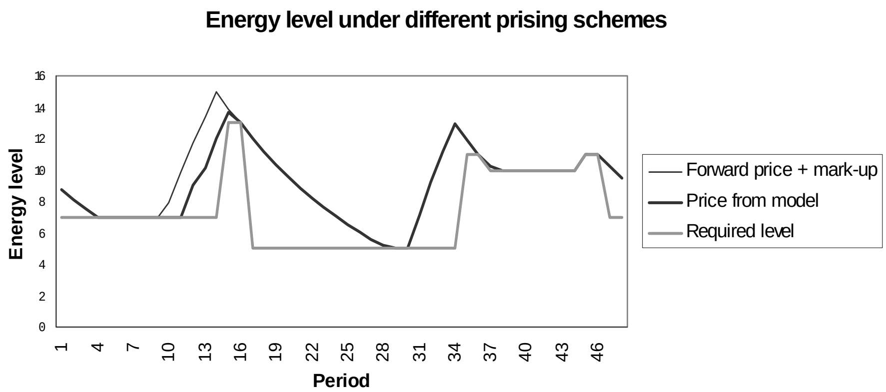


Figure 4: The picture shows how the energy level developed under the two different pricing schemes. In this case, the end-user starts warming up the house slightly later when offered the price profile from the Stackelberg solution than when offered the forward price plus a mark-up.

| **Pricing period** | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| **Price** | 71.4 | 77.1 | 48.0 | 70.3 |

Table 2: The table shows the results from the Stackelberg game with the day divided into four pricing periods. Pricing period 1 is from 11pm to 6am, period 2 is from 6am to 9am period 3 from 9am to 5pm and period 4 from 5pm to 11pm.

# 4 Conclusion

In this paper we developed a plausible model for domestic demand-side response to prices that vary over the day. The model shows how end users could save money by utilising the price profile. Real time metering also gives the retailer the opportunity to influence the end users' load profile through the pricing of power. To study the interaction between a retailer and an end user we developed a Stackelberg game model. The model is a mathematical program with equilibrium constraints (MPEC) and such models are often difficult to solve. In the present model the difficulties have shown up as a severe numerical instability.

# Acknowledgement

The author has benefited greatly from the helpful comments and suggestions given by Dr. Andrew B. Philpott. This paper was written while the author was a Visiting PhD Student at the Department of Engineering Science, University of Auckland and the hospitality of that department is gratefully acknowledged. I am also grateful to The Norwegian Electricity Association and The Research Council of Norway for their financial support.

# References

[1] H. Asano, H. Kakimoto, I. Matsukawa, *Household Response to Incentive Payments for Load Shifting: A Japanese Time-of-Day Electricity Pricing Experiment,* The Energy Journal, 2000, Volume 21, Number 1.

[2] M. Filippini, *Electricity demand by time of use. An application of the household AIDS model,* Energy Economics, 1995, Vol. 17, No. 3, pp 197-204.

[3] Z.-Q. Luo, J.-S. Pang and D. Ralph, *Mathematical Programs with Equilibrium Constraints,* Cambridge University Press, 1996.

[4] http://www.intelli.no/index_en.htm. The English version of the web pages of Intelli AS, January 2000.

[5] F. Tin-Loi and N. S. Que, *Nonlinear programming approaches for an inverse problem in quasibrittle fracture,* submitted to International Journal of Mechanical Sciences June 2001.