Electricity Network Reliability Optimization

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Abstract

Electricity distribution networks are subject to random faults. On occurrence of a fault in the distribution network, the circuit breaker on the faulty feeder trips, which disconnects supply to all customers connected to that feeder. Once the fault is located, the faulty component is isolated and then repaired. The cost of isolating and repairing faults depends on the way the network is configured, and the size and location of its components, since a major part of this cost comes from compensating customers if a fault in the network causes the interruption duration to be greater than a given threshold duration. We describe a model developed in collaboration with a local lines company for minimizing the cost of reliability of distribution networks by switch reconfiguration. Three local search heuristics to reconfigure radial distribution networks are investigated. For each component, the time between failures, the isolation duration, and the repair duration are modelled using Weibull distributions. The model allows the company to reconfigure their current radial distribution networks to minimize their total expected cost of reliability, as well as providing a tool for investigating capital investment scenarios.

1 Introduction

The power distribution network can be considered as two sub networks. The high voltage sub transmission network connects the Transpower national transmission network at the grid exit points to zone substations, at 110, 33 or 22kV. Each substation serves a particular geographic area. At the substations the voltages are further stepped down using transformers to 11kV or 6.6kV [1]. The function of the distribution network is to deliver electricity from the zone substations to customers. It includes a system of cables and overhead lines known as feeders operating mainly at 11kV, with some 6.6kV, which distribute electricity from the zone substations to distribution substations. At the distribution substations the voltage is stepped down to 400V and delivered to customers either directly or through overhead lines and cables.

The distribution system consists of interconnected radial circuits originating from zone substations. Distribution networks are normally operated in a radial configuration for effective coordination of their protective systems. More precisely, the radial circuits are connected to the substations via a circuit breaker that disconnects the feeder in case of a fault. Supply can be restored from alternative sources via the interconnections using switching operations.

1.1 The Reliability of Supply

The reliability of supply is governed by the frequency and the duration of interruptions to supply and the number of customers affected by interruption. On an occurrence of an electrical fault, the corresponding feeder's circuit breaker opens at the substation. This disconnects the entire feeder from supply of electrical energy. Consequently all customers on the faulted feeder experience an interruption in service. In most cases the fault is detected by the operations control systems or by "trouble calls" from customers, but in some cases a crew has to be dispatched to locate the fault. Once the fault is located, it is isolated by separating it from the rest of the network before restoring service to the unfaulted section. The faulty component can then be repaired. Subsequently the distribution system returns to its normal operating state.

1.2 The Cost of Reliability

In general, the total cost of isolating and repairing faults depends on the way the network is configured, and the size and location of its components. Increasing the expenditure on maintenance or extra capital can lower this cost by reducing the frequency and isolation time of faults. For a set of components with a given frequency of fault occurrence, it is possible to make further reduction in cost by configuring the switches in the network. Different configurations will not alter the type of faults that need to be repaired but may reduce the number of customers affected due to a particular fault. This will decrease the *cost of reliability*, or more precisely the monetary compensation by the distribution company to their customers if the interruption duration due to isolation or repair of a failed component is greater than a specific time interval called the threshold duration.

1.3 Distribution Network Reconfiguration

The reconfiguration of distribution networks changes a pair of switches to maintain the radial structure of the distribution network. In more precise terms, during reconfiguration, a normally open (NO) switch is closed. This action violates the radial network constraint by forming a loop. To redress the violation, a normally closed (NC) switch contained in the loop formed, is opened. The result is a newly configured radial network. The concept of reconfiguration is further explained in section 3.

For a large power distribution radial network, the number of possible configurations is extremely large. To find an optimal configuration with minimum total expected cost of reliability, a complete enumeration of the possible configurations would have to be done. This would consume extremely large amounts of time (it is an NP-hard problem) and thus this approach is very impractical. To overcome the time consuming process of complete enumeration, heuristics are employed in this model to reconfigure networks to get close to optimal solutions. This is further discussed in section 3.

2 Modelling The Radial Power Distribution Network

A radial network is equivalent to a spanning tree. Figure 2.1 shows a diagram of a radial distribution network.

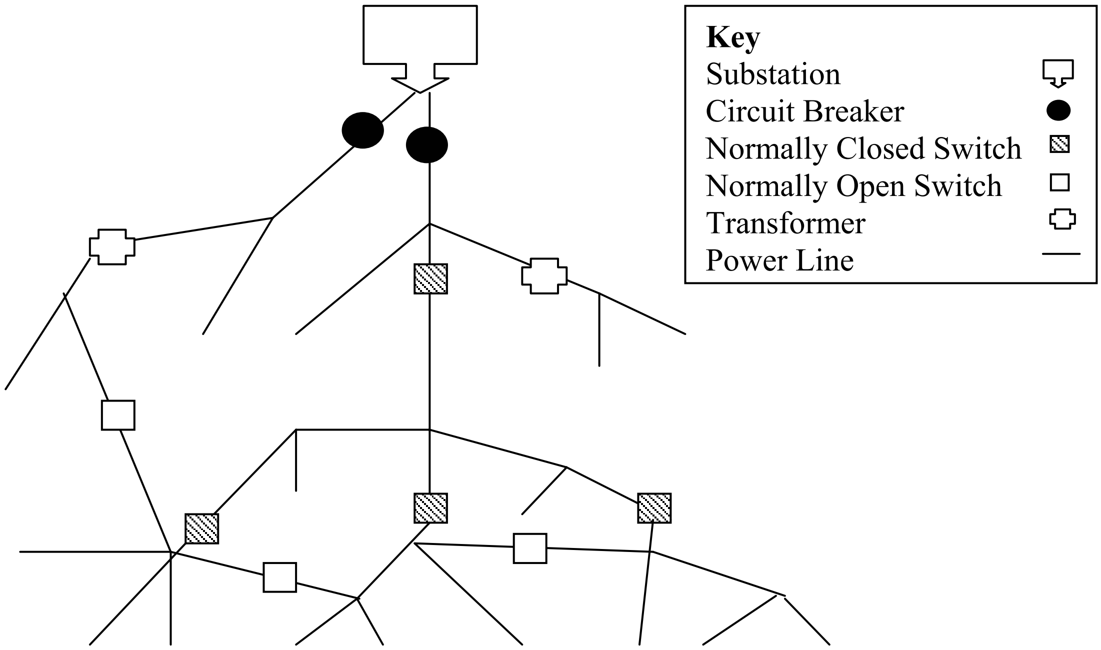

Figure 2.1 A radial distribution network

The radial network configuration is modelled as a graph, where:

- The edges represent *switches*
- The vertices represent collections of power systems gear containing no switches, which we shall henceforth refer to as a "*component*".

Each component comprises power systems gear such as power lines and cables, transformers, power poles, fuses etc. The switches (labelled according to their location, function and status) represent the NC switches, NO switches and Circuit Breakers. Each component consists of all the power systems gear between adjacent switches. When clumping power systems gear into a component, it is assumed that upon failure of any power systems gear within the component, all the power systems gear represented by that component and all the consumers connected to the component would be disconnected during isolation and repair.

2.1 Fault Effect – Overview

When there is a fault in the radial network, the closest circuit breaker upstream of the fault trips and disconnects the entire network downstream of the circuit breaker. It is assumed that the network remains disconnected until the fault is isolated. Then power is restored to all the possible areas of the interrupted network by opening and closing switches. All the components in the isolated area of the network remain disconnected until the faulty component is repaired. In general most power distribution companies have to compensate their customers if the interruption duration due to isolation or repair of a failed component is greater than the threshold duration. In our model, the isolated area may consist of a single component in which the power systems gear has failed, or include several components that are directly or indirectly linked to the failed component while having no other switches connecting them to the rest of the network thus restricting their restoration.

2.2 Formulation of The Stochastic Model

The time between failures, the isolation duration, and the repair duration involve uncertainty. Therefore these durations need to be modelled using a stochastic model. We approximate these stochastic durations using the Weibull distribution.

The failure frequency for a component is the number of times the component fails over a period of time. A failure for a component can be described as transitioning from an operating (Up) state to an isolation state. The probability of this transition occurring is approximated using a Markov process. It can be seen that the component state probabilities are independent of the embedded Markov Chain stationary state probabilities. Using the component state probabilities, the state frequency *Fr(j)* for component *j* is

$$Fr(j) = \frac{1}{(\mu_u + \mu_i + \mu_r)},$$

where μ is the Weibull mean duration and subscripts u, r and i represent the component 'Up', 'Isolation' and 'Repair' states respectively. It can be seen that the state frequency is the same for all component states.

In computing the expected cost of reliability, the probabilities for the isolation or repair duration being greater than the threshold duration need to be determined. We assume that during isolation and repair of a failed component in a power distribution network, no other component fails. This assumption is justifiable when failures are independent and occur with small probability.

Let:

I = Isolation Duration, R = Repair Duration, Q = Threshold Duration.

We assume that I, R and Q each have a Weibull distribution. The Weibull Distribution is defined by the following probability density function and cumulative distribution function:

$$f(t) = \frac{\beta}{\eta^{\beta}} t^{\beta-1} e^{-\left(\frac{t}{\eta}\right)^{\beta}}, \qquad F(t) = 1 - e^{-\left(\frac{t}{\eta}\right)^{\beta}}.$$

Probability that the isolation duration is greater than the threshold duration is:

$$\Pr(I > Q) = 1 - F_i(Q) \;= e^{-\left(\frac{Q}{\eta_i}\right)^{\beta_i}}.$$

Probability that the isolation duration is less than the threshold duration, but the isolation + repair duration is greater than the threshold duration is:

$$\Pr(I < Q \cap I + R > Q).$$

The distribution of I+R is given by a convolution integral:

$$\Pr(I < Q \cap I + R > Q) = \int_0^Q f_I(t) \left[\int_{y=Q-t}^{\infty} f_r(y) dy \right] dt$$

$$= \int_0^Q f_i(t) \cdot e^{-\left(\frac{Q-t}{\eta_r}\right)^{\beta_r}} dt .$$

Substituting the probability density function $f_i(t)$, gives

$$\Pr(I < Q \cap I + R > Q) = \int_0^Q \frac{\beta_i}{\eta_i^{\beta_i}} t^{\beta_i - 1} e^{-\left[\left(\frac{t}{\eta_i}\right)^{\beta_i} + \left(\frac{Q-t}{\eta_r}\right)^{\beta_r}\right]} dt \ . \tag{1}$$

2.3 Computing the Cost of Reliability

Using the failure frequencies and the duration distributions derived above, it is simple to compute the total expected cost of reliability (COR) for a given radial configuration.

Let:

$R_C(i)$ = expected cost of reliability due to failure at component i;

$C_I(i)$ = cost of compensating customers when the time to isolate a fault at component i is greater than the threshold duration;

$C_{IR}(i)$ = cost of compensating customers when the time to isolate a fault at component i is less than the threshold duration, but the isolation plus repair time is greater than the threshold duration;

$Fr(i)$ = frequency of failures at component i;

$P_I(i)$ = $\Pr(I > Q)$ = probability that the time to isolate a fault at component i is greater than the threshold duration;

$P_{IR}(i)$ = $\Pr(I < Q \cap I + R > Q)$ = probability that the time to isolate a fault at component i is less than the threshold duration, but the isolation plus repair time is greater than the threshold duration.

Then the total expected COR is computed as:

$$\sum_{i=1}^{N} R_C(i) = \sum_{i=1}^{N} Fr(i)[C_I(i) \cdot P_I(i) + C_{IR}(i) \cdot P_{IR}(i)]$$

where N is the total number of components. The calculation of the expected COR is straightforward except in the case of the term $P_{IR}(i)$ which is the convolution integral (1). In practice we compute this numerically using Simpson's rule.

3.0 Radial Distribution Network Reconfiguration

A power distribution company wants to configure their distribution network to minimise the expected cost of reliability. To determine the configuration that gives the minimum expected cost of reliability; all possible radial configurations have to be evaluated to determine the best configuration. The computational time taken to completely enumerate all possible configurations for a large network would be very long. Hence a complete enumeration approach is impractical, and therefore we use a local search approach.

3.1 Local Search Heuristics

Local search heuristics are employed to reconfigure radial networks to give a local minimum for the total expected cost of reliability. The configuration that gives a local minimum could be a configuration that gives a global minimum, but this is not guaranteed. A local search solution is not guaranteed to be a global minimum, because a local search heuristic does not explore all possible configurations; consequently a possible configuration not evaluated could be the global minimum.

A local search heuristic is a 'neighbourhood search algorithm' that explores neighbouring solutions (radial network configurations). The solutions explored depend on the neighbourhood rule implemented and how the neighbourhood structure is defined. The neighbourhood of a particular radial network configuration is the set of all radial network configurations that can be obtained from the particular radial network configuration using the operations set out by the neighbourhood rule. A further characteristic of a local search heuristic is its descent rule. The descent rule determines the neighbouring solution that is to be accepted out of all the possible neighbouring solutions defined in the neighbourhood structure.

The following is some notation used to describe the local search neighbourhood structures and the rules used in this model.

- An 'Entering Switch (ES)' is a NO switch that we select to become a NC switch.
- A 'Leaving Switch (LS)' is a NC switch that we select to become a NO switch.

A general neighbourhood search algorithm that can be used to reconfigure a radial network is as follows. An ES is selected for the current radial network configuration. Closing this switch forms a loop (cycle in the spanning tree) in the network that violates the necessary radial constraint. To restore the network back to a radial configuration, a LS has to be chosen from the loop formed. Observe that the choice of potential LS's depends on the ES chosen, though we suppress this dependence in the notation. As an example, consider the radially configured network in Figure 3.1 called RN-0.

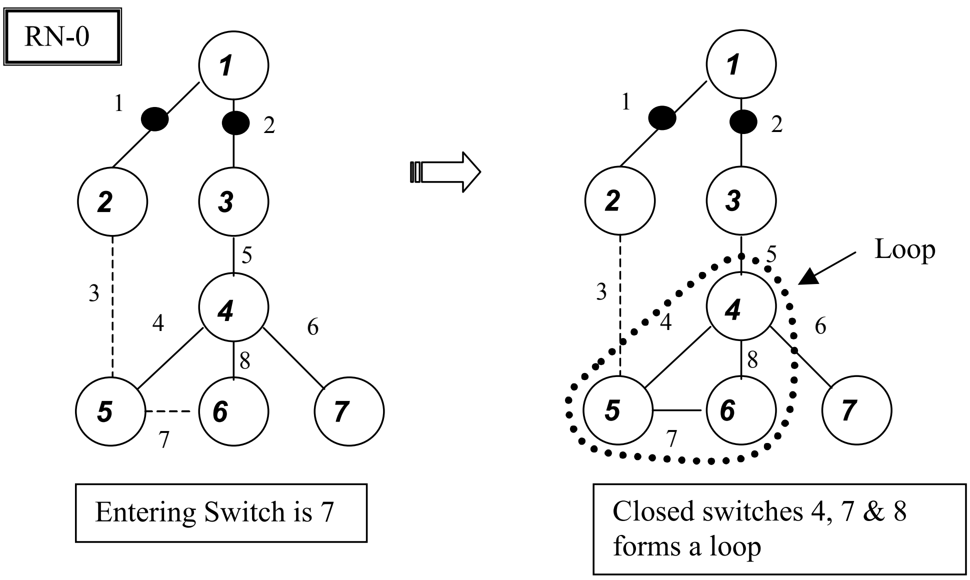

Figure 3.1 The initial stage of reconfiguration where switch 7 is selected as the ES from RN-0. This forms a loop that violates the radial network

To restore the network to radial configuration, a leaving switch (other than the ES 7) in the loop has to be opened. Suppose switch 4 is chosen to be the LS. Figure 3.2 shows the next step of reconfiguration resulting in a new radially configured network RN-1.

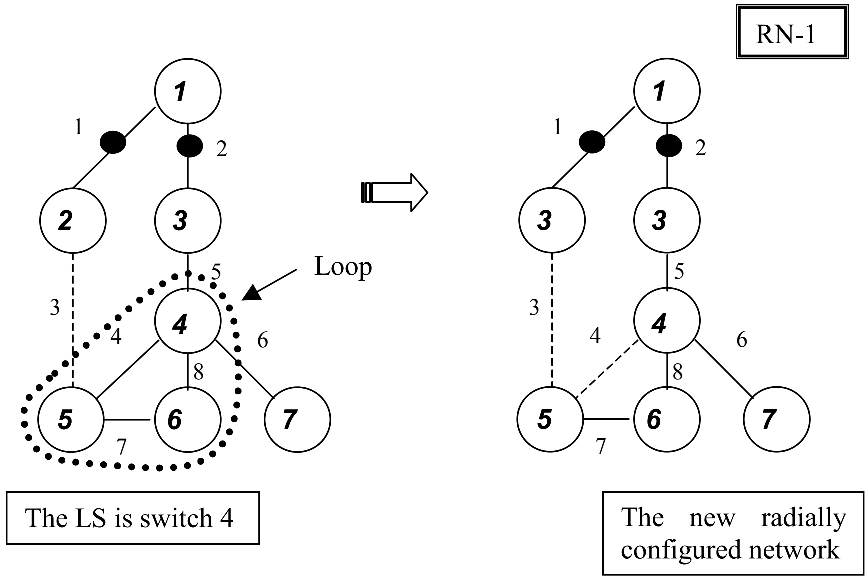

Figure 3.2 The second stage of reconfiguration where a switch from the loop is nominated as the LS to give radial network RN-1

In the following sections we describe three heuristics: the Adjacent Neighbourhood Steepest Descent Search, the Cycle Neighbourhood Steepest Descent Search and the Cycle Neighbourhood Next Descent Search.

3.1.1 Adjacent Neighbourhood Steepest Descent Search

In the Adjacent Neighbourhood (AN) search, the neighbourhood structure of the current radial network is the set of all radial networks that can be obtained by the operation of closing a NO switch to form a cycle and opening either one of the two NC switches in the cycle adjacent to the NO switch, so as to produce a radial network.

The descent rule utilized in the local search heuristic for the AN structure is the steepest descent rule. The AN steepest descent rule takes the following approach. For any incumbent radial network, we evaluate the total expected cost of reliability (COR) of all solutions in the adjacent neighbourhood, and select that which has lowest total expected COR, (if that gives an improvement) to be the incumbent. These searches are done repeatedly until there is no solution found which gives a lower total expected COR than the incumbent solution. At this point the incumbent solution is declared to be the local minimum.

3.1.2 Cycle Neighbourhood Steepest Descent Search

In the Cycle Neighbourhood (CN) search, the neighbourhood structure of the current radial network is the set of all radial networks that can be obtained by the operation of closing a NO switch (from the set of possible ES's) to form a cycle (set of possible LS's), and then selecting a LS from the set of possible LS's, so as to produce a radial network.

The descent rule utilized in the local search heuristic for the CN structure is the steepest descent rule. The CN steepest descent rule takes the following approach. At each search by the CN steepest descent local search heuristic, each ES from the set of ES's is considered. Subsequently for each ES, the total expected COR is determined for all the radial networks formed when considering the corresponding LS's one-by-one. Out of all the configurations explored in a single search, the ES and LS combination that gives the minimum total expected COR is accepted as the incumbent solution. The incumbent solution is stored and then another steepest descent local search starts with the radial network configuration given by the incumbent solution as the initial radial network. These searches are done repeatedly until there is no solution found which gives a lower total expected COR than the incumbent solution. At this point the incumbent solution is declared to be the local minimum.

3.1.3 Cycle Neighbourhood Next Descent Search

The 'Cycle Neighbourhood Next Descent' local search has exactly the same neighbourhood structure definition as that of the CN steepest descent local search. The difference is in the descent rule employed. Given a neighbourhood rule with a CN structure, a local search heuristic with the 'Next Descent' rule evaluates neighbouring solutions (radial configurations) and accepts the first solution found which gives a lower total expected COR then the incumbent solution. The next descent rule is also referred to as the 'First Improvement' rule.

The CN next descent rule takes the following approach. First an ES is selected along with a LS. This forms a new radially configured network for which the total expected COR is computed. If the total expected COR is less than that for the incumbent (initially set to be the total expected COR of the initial network), then we accept the new radially configured network as the incumbent solution. Otherwise we consider the next LS from the set of LS's to form a new radial configuration. If all LS's from the set have been considered, we then consider the next ES from the set of ES's and thus a LS from its corresponding set of potential LS's, to form a new radial configuration.

A single search of the CN Next Descent local search consists of the aforementioned steps. Every search starts with the most recently accepted radial network configuration (i.e. the incumbent solution) as the initial radial network from which the sets of ES's and LS's are determined in accordance with the defined neighbourhood structure. These searches are done repeatedly until there is no solution found which gives a lower total expected COR than the incumbent solution. At this point the incumbent solution is declared to be the local minimum.

3.2 Modelling Network Load Capacity

While it is important for a power distribution company to have their radial networks configured for minimum total expected cost of reliability, it is also essential that the reconfigured radial networks do not exceed the load capacities of the network components. When reconfiguring the radial network, each reconfiguration has to be checked to ensure that the load capacity is not exceeded at any part of the network. One approach is to carry out a load flow analysis, using power systems principles to calculate the power flows and voltages of the power system under normal operation conditions.

We take a simpler approach to model the load capacities of the network. We assume that the clumped power system gear represented by a node in the radial network has the capacity to supply all the customers connected to it and all the power system components in successor nodes. We specify a load capacity for each switch, and check that this is not violated when a configuration is chosen. In the current version of our model no load capacity check is carried out for the resulting network configuration during repair, although this feature can be added without any essential changes.

4.0 Results and Conclusions

Using the models and concepts developed in the project, a Visual Basic spreadsheet-based tool called the Network Reliability Optimizer (NeRO) has been developed. Once all the data defining the components and the switches have been entered into NeRO, it can be executed. The program uses the graph definition to randomly generate a radially configured network. Randomly generated radial networks that meet the load feasibility criteria, allows each local search heuristic to have different starting configurations, thus increasing the chance that a global minimum will be found. Using the randomly generated radial network as an initial configuration, the program subsequently computes the total expected COR and stores the network as the incumbent solution. Finally the program starts the local search heuristic.

4.1 Case Study of Takanini17 Feeder

A case study was carried out on the Takanini17 feeder. This feeder is part of VECTOR Ltd distribution network and has a poor performance history. VECTOR Ltd is investing in a power restoration link (PRL) at the end of the Takanini17 feeder from a neighbouring distribution company. Using NeRO, the total expected cost of reliability was compared for the current configuration with and without the PRL. Analysis with the model indicated a significant decrease in the total expected cost of reliability, hence supporting the investment decision. Further, the feeder was optimally configured with the PRL investment, to give an even lower total expected cost of reliability.

4.2 Performance Comparison of the Local Search Heuristics

To analyse the performance of the local search heuristics developed in this project, a large network was used. The network had 33 components and 62 switches. All the data that was required to define a complete network was randomly generated. Figure 4.1 shows how each local search heuristic would perform for 300 runs given that every solution accepted at each run would be no worse than the solution found in the preceding run. From this perspective, the plot for each local search heuristic shows its convergence characteristics. It can be concluded that the Adjacent Neighbourhood Steepest Descent local search heuristic is the best performing heuristic as it finds a local minimum that is no worse than that found by the Cycle Neighbourhood Steepest Descent and the Cycle Neighbourhood Next Descent local search heuristics, in the shortest time period.

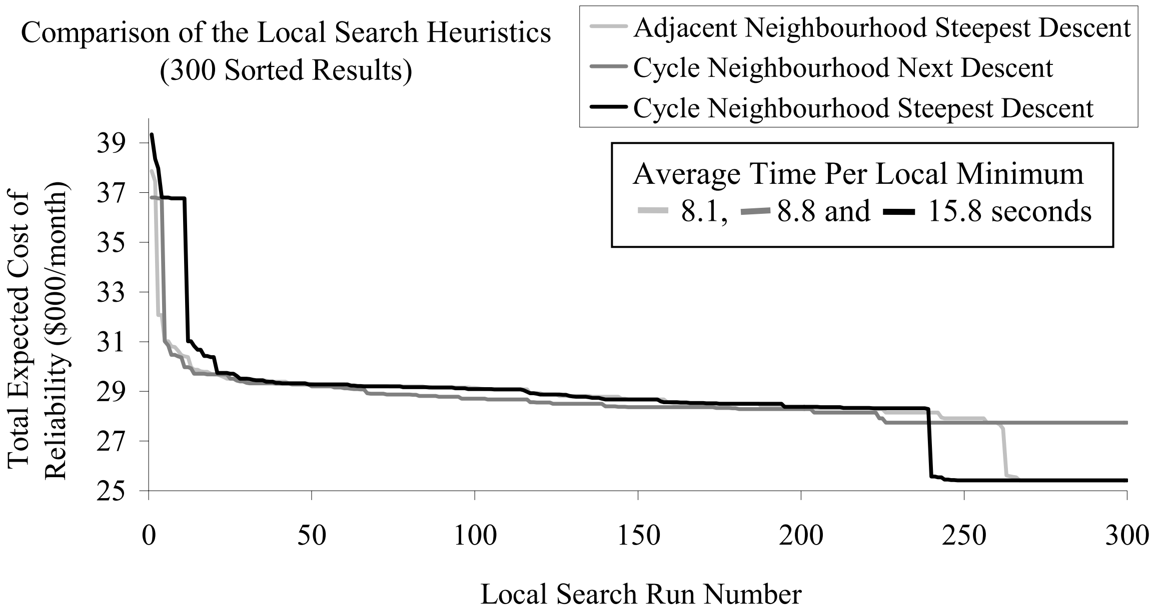

Figure 4.1 Comparison of the convergence characteristics of local search heuristics

NeRO enables VECTOR to reconfigure their current radial distribution networks to minimise their total expected cost of reliability. Additionally NeRO can be used as a planning and design tool to investigate capital investment scenarios. It can be enhanced to carry out a full load flow analysis using power systems concepts and provides a good foundation for development of programs to minimise power loss, and optimise capacity usage in conjunction with minimising the cost of reliability.

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References

[1] Asset Management Plan (2000/2001). VECTOR Limited. Auckland, NZ.