

A Tabu Search Heuristic for the Inventory Routing Problem

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Abstract

In the Inventory Routing Problem, a distributor must choose the size and frequency of deliveries to each of its customers and design vehicle routes so as to avoid stockouts over a given planning horizon. We propose a tabu search heuristic for the deterministic version of this problem. Computational results are reported on randomly generated test problems.

1 Introduction

Nowadays, companies are keen to invest resources to increase productivity. The most common objective is to save on costs; for distribution companies the transportation costs are the perfect target. New information management technology, combined with sharp resolution methods, can lead to great savings.

For the Vehicle Routing Problem (VRP), a fleet of vehicles is available at a distribution centre. These vehicles need to visit many delivery points while satisfying their capacity and time constraints. This problem is NP-hard, hence it is extremely difficult to find the optimal solution. However, heuristics can find good solutions in a reasonable time. If the objective consists of minimizing long term costs, the problem should be resolved considering the impact of the solution on future costs. Thus determining the length of the resolution period is critical. When inventory constraints are added to the VRP, the problem is known as the Inventory Routing Problem. We propose a new heuristic to find solutions that minimize the transportation costs, while respecting the customers demand. The industry applications of the IRP are numerous, for example industrial gas, propane and food distribution as well as in blood collection.

The objective of this paper is to propose a Tabu Search heuristic for the resolution of the Inventory Routing Problem (IRP). The Tabu Search program of Cordeau, Gendreau & Laporte (1997) for the Periodic Vehicle Routing Problem is modified for the IRP. We use the assumptions of deterministic and known demand. Finally, instances are randomly generated to test the heuristic.

2 Inventory Routing Problem

The IRP consists of a repetitive distribution of a product from a single depot to multiple customers while preventing stockouts on a predetermined time horizon. The routes are established for a predetermined number of days. For example, consider a distribution company that has the technology to evaluate the daily consumption rate of its products by its customers. This company can now use its own decision-making logic for all the logistics issues. This way, customers would not need to make orders anymore. The distribution company would have to take into account the inventory level to decide when to visit them and how much to deliver. The distance between the customer and the depot is a key element. If the objective is to minimize transportation costs, far customers would not be visited often. Precise information at any time on the inventory levels and location is required in order to take cost-effective decisions for the company and its customers.

The Inventory Routing Problem is a multi-period problem. For every day of the period the program needs to resolve three sub-problems. This issue is very complex because the solutions of the sub-problems are interrelated:

- Determine the customers to visit
- Determine the quantity to deliver
- Design the vehicle routes

The global optimization of these sub-problems creates better results than fixing the solution of one problem before resolving the second problem. The combined modelling of these problems is complex because the solutions of the two first problems (the customers to visit and the quantity to deliver) is the input for the Vehicle Routing Problem.

Golden, Assad & Dahl (1984) discussed the difficult problem resulting from the addition of the inventory constraint to the Vehicle Routing Problem : *“The multi-period nature of the problem is largely responsible for its complexity. Since decisions taken on different days interact, myopic decision-making can cause serious problems [...]”* .

Therefore, it is extremely important to choose a resolution method for the Inventory Routing Problem that is sensitive to this difficulty. The resolution of the IRP is managed through the addition of some simplifying assumptions. For example, the customers' consumption rate can be considered as deterministic or stochastic. This research is based on a deterministic consumption rate. The problems are also separated between those with a single delivery per period and those which can assign a frequency of deliveries to customers. This project considers the possibility for a frequency of deliveries since it is more interesting and realistic, but it implies more interactions between the sub-problems.

3 Methodology

The Vehicle Routing Problem is very well known and many classical heuristics have been developed. However, the classical heuristics success is quite limited. They give a good solution in a reasonable time, but can not guarantee optimality. Indeed, when a local minimum is reached, the procedure finishes. The resolution is stopped by a local optima. In order to extend the exploration of the possible solutions, a notion of memory is used by the Tabu Search heuristic. It keeps track of all the solutions that have already been visited.

The Tabu Search is a metaheuristic that allows degradation of the solution to overcome local optimum by temporarily accepting non realisable solutions. The metaheuristics are promising, they combine the notions of construction, memory and improvement of the solution. The neighbourhood search for the current solution is determined by the evolution of the past search. An improvement heuristic is then used to move from one solution to the other.

3.1 Tabu Search

Tabu Search allows the degradation of the solution combined with the notion of memory to avoid cycling. By this procedure, it is possible to overcome a local optima. Therefore, considering the current solution s , the next solution s' in the neighbourhood of s is chosen without reference to the cost of s . If s was a local optimum, it is likely that the next move from s' will be to go back to s . The solution could cycle between the two solutions.

To avoid cycling, the transformation leading to this solution is memorized. The reverse transformation is then prohibited for a certain number of iterations. Since the reverse exchanges are tabu, cycles up to a certain length are avoided.

A solution neighbourhood is composed from a certain number of admissible solutions obtained by a given transformation. For the IRP, the transformation can be to swap customers between routes, days of visit or combinations of visits (these are the attributes of a solution). To modify the attributes of a solution, we used the GENI algorithm. It has been developed by Gendreau, Hertz & Laporte (1992). This method removes and inserts a customer between the other customers of a route, consecutive or not, while locally reoptimising the route. The results obtained by this method show its effectiveness.

Different elements need to be defined to develop a Tabu Search algorithm for the Inventory Routing Problem :

- Solution space
 - o Contains all possible routes visiting all customers for each of their combination of visits
 - o Overcapacity and overduration are allowed with penalties dynamically adjusted
- Neighbourhood
 - o Exchanges of the combinations of visits
 - o Route exchanges
 - o Use of GENI for a local optimization
- Tabu principle
 - o List of attributes of a transformation (a customer on a route a certain day)
- Aspiration criteria
 - o Revoke a tabu when the cost of a solution is better than the best known cost
- Continue diversification of the search
 - o Penalty on an attribute according to its frequency
- Stopping criterion
 - o After a fixed number of iteration

3.2 Sensitivity analysis

Before running the algorithm on literature instances, it is important to carefully calibrate the following parameters:

- Stopping criteria : number of iterations
- Dynamic adjustment of penalty weights for overcapacity and overduration
- Diversification intensity
- Length of tabu list
- Neighbourhood size for GENI insertions

A series of 10 instances has been randomly generated to test different values of the parameters. The parameters that improve the efficiency of the algorithm with a good compromise between computation time and solution quality are chosen. The values obtained with these test instances are then used to solve the literature instances.

3.3 Combinations generation

For each customer, we generated all the possible combinations of visits. A combination is a boolean value associated to each day of the period indicating if the customer is visited. For example, on a 5 day period a combination could consist of only one visit on day 1 or visits on days 2 and 4.

The global resolution method for a rolling horizon of the Inventory Routing Problem is the following:

- Generate legal combinations for two periods
- Solve and implement only the first period
- Start the procedure again for the next two periods

This results in a long term resolution horizon while solving only on a short term basis.

Resolving the Inventory Routing Problem is time consuming. To improve the computation time, a reduced group of visit combinations is attributed to each customer. Different rules have to be followed in order to attribute only combinations that are interesting for the algorithm on the premise of the rolling horizon:

- No stockouts
- Legal spacing out
- Keeping only smallest delivery frequency

A combination of visits that assures a double period without stockouts is essential. It allows a long term horizon. Since a combination where a customer is visited two days in a row and never after in a five day period is not likely to be part of the optimal solution. The legal spacing out allows us to ignore these combinations and thus reduce the computation time. It is natural to think that if we can visit a customer only twice on a period, this will reduce the transportation costs compared to daily visits. By keeping only the smallest delivery frequency, we drastically reduce the number of combinations.

4 Instances

Instances were generated from those of Chao, Golden & Wasil (1995). The structure of the customers has been preserved, but new particularities have been added. The required information to run the algorithm are the following:

- Vehicles characteristics
 - o duration
 - o capacity
- Customers characteristics
 - o coordinates
 - o capacity
 - o starting inventory
 - o daily consumption rate

On these new instances, the stopping criteria was set at 30 000 iterations. For 48 customers and a period of 4 days, it took 3.10 CPU minutes. For 288 customers and a period of 6 days, we needed 35.36 CPU minutes. The instances and their solution are available for future research comparison.

We also used twelve instances of Bertazzi, Paletta & Speranza (2002). They resolved a 30 day period for 50 to 100 customers with only one vehicle. The period of 30 days caused us a problem, because our algorithm is implemented on a rolling horizon basis which is not the case for Bertazzi, Paletta and Speranza (BPS). Since most of their solutions only used the vehicle every other day, we resolved the instances on a period of 15 days while doubling the daily consumption rate. Our CPU computation time was greater but for a problem resolved once every 30 days, this calculation time is acceptable. The Tabu Search obtained cheaper transportation costs than BPS only on the 12th instance. However, the costs are on average only 1.58% greater than BPS, which is relatively good. The solution obtained by BPS on the instances 5, 7, 8, 9 and 12 offers visits to customers exactly every other day. These solutions are then fully comparable with our solutions and we obtained a cost difference of only 0.40%.

This difference comes from the fact that the Tabu Search is not used at its full potential on the BPS instances. They used only one vehicle to make the deliveries and this handicapped our method because no exchanges could be made between routes. This exchange is extremely important in the heuristic process to improve the solution in place by calling the GENI procedure. Hence, we did not have a route optimization anymore. In reality, we usually have a fleet containing more than one vehicle. Furthermore, this vehicle had an infinite day duration and capacity. Therefore, our parameters that adjust the penalty for overduration and overcapacity were obsolete. By adding limits to the duration of a route and the capacity of a vehicle, the problem would be more realistic and the resulting routes would be similar for one day to the next. Hence, the BPS instances are not complex enough to adequately test the efficiency of the Tabu Search.

5 Conclusion

A Tabu Search algorithm was presented to resolve the Inventory Routing Problem. This method was found to be flexible and efficient on multi-customers and multi-vehicles instances. On the other hand, the actual implementation is limited by the length of the period. This could be resolved by future research.

The algorithm has been used to resolve the adapted instances of Bertazzi, Paletta & Speranza (2002). The results are not exactly comparable, but the costs obtained are relatively similar (1,58% over in average). The instances were not complex enough to fully benefit from the flexibility of our method. Even though the Tabu Search heuristic is a more general method it still obtains good results on these more simple problems.

Finally, new instances have been created and resolved in order to overcome the lack of appropriate case studies in the literature. Moreover, the instances of Chao, Golden & Wasil have been modified for the Inventory Routing Problem. The instances and their solution are available for future research.

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7 References

- Bertazzi, L., G.Paletta, and M.G. Speranza, 2002. "Deterministic order-up-to level policies in an inventory routing problem.", *Transportation Science* **36**:119-132.
- Chao, I-M., B.L. Golden, and E.A. Wasil. 1995. "An improved heuristic for the period vehicle routing problem." *Networks* **26**:25-44.
- Cousineau-Ouimet, K., J.-F. Cordeau, and G. Laporte. 2002. "Une méthode de recherche avec tabous pour le problème de tournées de véhicules avec contraintes de stocks." Master thesis, Department of Quantitative Methods, École des Hautes Études Commerciales, Montreal, Canada.
- Cordeau, J.-F., M. Gendreau, and G. Laporte. 1997. "A tabu search heuristic for periodic and multi-depot vehicle routing problems." *Networks* **30**:105-119.
- Gendreau, M., A. Hertz, and G. Laporte. 1994. "A tabu search heuristic for the vehicle routing problem." *Management Science* **40**:1276-1290.
- Golden, B.L., A.A. Assad, and R. Dahl. 1984. "Analysis of a large scale vehicle routing problem with an inventory component." *Elsevier: Large scale systems* **7**:181-190.