Location of Rescue Helicopters in South Tyrol

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Abstract

South Tyrol is a popular destination in Northern Italy for tourists from the north and south of the Alpine mountain ranges. The growing demand for tourist activities such as skiing, hiking and climbing has led to a large number of accidents in the region over the years. This has resulted in an increased demand for rescue helicopters, as most of the accident sites are not accessible by vehicles on-ground. In this project we focus on optimising the locations of three rescue helicopters coordinated by a South Tyrolean rescue organisation called Weißes Kreuz in order improve the emergency response times.

Two models were formulated to solve this problem. The first model minimises the average response times and the second minimises the worst response times. We solve the two models using some effective heuristics. These heuristics do not guarantee optimality but do provide local optimal solutions that can improve the response times considerably. The methods implemented have shown that the model that minimises the worst response time results in more evenly distributed accident sites to the new helicopter base locations.

1 Introduction

This paper presents a study on the location of rescue helicopters in South Tyrol, Italy. The study focuses on a South Tyrolean rescue organisation called Weißes Kreuz that provides rescue facilities for all the accidents that occur within the region. The following sections describe the location problem in detail.

1.1 Background on The Weißes Kreuz and South Tyrol

Weißes Kreuz is a non-profit rescue organisation associated with Samaritan International and is operating in South Tyrol which is situated in the northern part of Italy. It coordinates three rescue helicopters. Two of which, named Pelikan 1 and Pelikan 2, are owned by the Weißes Kreuz. The third one is called Aiut Alpine Dolomites (AAD in remaining text) and is operated by the mountaineering rescue organisation in the Dolomites region of South Tyrol. A single call centre answers the emergency calls and also dispatches the three helicopters [1].

South Tyrol is a popular destination for tourists from the north and south of the Alpine Mountain ranges. The growing demand for tourist activities such as hiking and climbing in the summer and skiing in winter has led to an increased number of accidents in recent years. Due to the mountainous terrain, many of the accident sites are not accessible by cars or vans which has therefore led to an increased demand for rescue via helicopters [1].

1.2 Problem Description

Currently, the first rescue helicopter, Pelikan 1, is stationed in Bozen, the capital city of South Tyrol, Pelikan 2 is stationed in a town called Brixen and AAD is stationed in Kastelruth as depicted in Figure 1.

The problem with the current situation is that the three helicopters are stationed too close together in the central part of South Tyrol. Pelikan 1 is stationed too far in the east of the region and Pelikan 2 is stationed too far in the west of the region. This worsens the emergency response times.

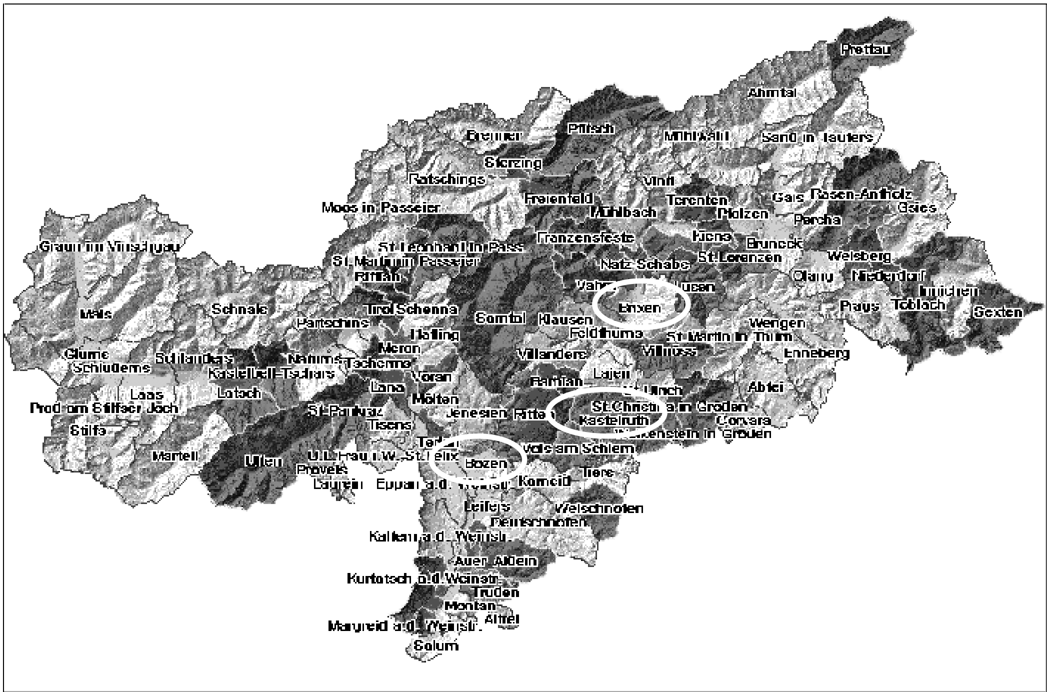

Figure 1: Map of South Tyrol [2]

1.3 Focus of Project

The aim of this project is to find ways to improve or optimise the current base locations of the helicopters coordinated by Weißes Kreuz, in order to shorten the emergency response times. For this project, we take into consideration that the bases can be located anywhere in the region as long as they are accessible and can be staffed permanently.

We have used various heuristics, algorithms and mathematical techniques to find solutions to the base location problem. Although these methods may not guarantee optimality, they can provide close-to-optimal solutions that reduce the emergency response times significantly.

1.4 Data

The data provided by Weißes Kreuz consisted of a list of 1928 missions flown over the years 1996 to 1997 [1]. We were given the coordinates of 109 accident sites and 3 helicopter bases in the South Tyrol region where the coordinates had been measured using an official map of the South Tyrolean Statistical Institute. The location of the administrative centres of the towns in the area of which the accidents took place were taken to be the point representing each of these accident sites because the coordinates for the exact location of the accident site were not available [1]. We were also provided with the number of missions to the area of each town (see Figure 1). This defines the weighting on a particular accident site.

2 Mathematical Models

The criterion for finding a good base location for the helicopters requires the improvement of the response times to the emergency calls. The response times depend on the distances between the helicopter bases and the accident sites. This gives rise to two basic models, one that will improve the average response times by minimising the total weighted distance between the helicopter bases and the accident sites. And the second that will improve the worst response times by minimising the maximum un-weighted distance between a helicopter base and the accident site.

2.1 Model 1: Minimising the Total Weighted Distance to Improve the Average Response Times

This model finds p points $X_1, \ldots, X_p$ for the location of the new emergency facilities where $X_j = (x_j, y_j)$ such that the total weighted distance between the accident sites, i.e. the demand points at locations $P_1, \ldots, P_n$, and their closest new emergency facility is minimised. This model will minimise the average response times in the region since it takes the weighting into account and therefore is termed the "Weighted p-Median Model".

$$\min_{X_1,\ldots,X_p} \sum_{i=1,\ldots,n} w_i \min_{j=1,\ldots,p} d(X_j, P_i)$$

Since we are dealing with helicopters, the Euclidean distance was considered as being the realistic distance between a helicopter base and an accident site. The Euclidean distance is a measure of the straight line between two points; in this case the two points are the accident site and the base location. Therefore, in the mathematical model above $d(X_j, P_i)$ is the Euclidean distance between X_j (base location j) and P_i (accident site i) where:

$$d(X_j, P_i) = \sqrt{(x_j - Px_i)^2 + (y_j - Py_i)^2}$$

Finding the minimum of $d(X_j, P_i)$ is difficult because this is a non-convex optimisation problem. This is because we know that the Euclidean distance is a quadratic function and the minimum of many quadratic functions forms a non-convex function. Therefore the Weighted p-Median Model is hard [3].

2.2 Model 2: Minimising the Maximum Un-weighted Distance to Improve the Worst Case Response Times

This model finds p points $X_1, \ldots, X_p$ for the location of the new emergency facilities such that the maximum un-weighted distance between the accident sites, i.e. the demand points at locations $P_1, \ldots, P_n$, and their closest new emergency facility is minimised. This model will only minimise the worst response time in the region since it does not take the weighting into account and therefore is termed the "Un-weighted p-Centre Model".

$$\min_{X_1 \ldots X_p} \max_{i=1..n} \min_{j=1..p} d(X_j, P_i)$$

Once again, this model results in a non-convex function because the interior part of the model still requires finding the minimum $d(X_j, P_i)$, as was the case for Model 1, and so the Un-weighted p-Centre Model is hard, too [4]. For this reason, we have developed a heuristic to solve Model 1 and 2.

3 Heuristic Solution Approach

The applied heuristic allows us to allocate each of the accident sites to its nearest helicopter base. This helps avoid multiple allocations. We then form a subdivision by grouping all the accident sites that are allocated to a particular helicopter base together. Model 1 and 2 can then be solved by treating each of the subdivided regions as a separate problem.

Once the region is subdivided, Model 1 for each region j becomes:

$$\min_{X_j} \sum_{i=1,\ldots,n} w_i d(X_j, P_i) \quad (1)$$

and is termed the "Weighted Median Model" because it reduces the average response time by minimising the total weighted distance travelled by the three helicopters.

Model 2 for each region j becomes:

$$\min_{X_j} \max_{i=1..n} d(X_j, P_i) \quad (2)$$

and is termed the "Un-weighted Centre Model" because it reduces the worst response time by minimising the maximum un-weighted or the longest distance travelled by a helicopter. Here j ranges from region 1 to region 3 for the three helicopter bases. By solving Models (1) and (2) we obtain a new base location in each of the subdivided regions. We can then reallocate each of the accident sites to its nearest helicopter base to form a new subdivision and continue this process until there are no more changes in the base locations. We show how to solve Models (1) and (2) in Section 4 and 5.

4 Solution Approach for Model 1

4.1 Generating Starting Solutions Using the Squared Euclidean Distance

Using the Squared-Euclidean distance was the first step to finding a solution for the location of the three helicopter bases. Our model for this approach is:

$$\min_{X_j} \sum_{i=1,\ldots,n} w_i d(X_j, P_i)^2$$

We then differentiate the expanded form of the squared Euclidean distance and set it to zero to obtain a formula for *x* and *y* which gives us the coordinates of the helicopter base:

$$x = \frac{\sum_{i=1}^{n} w_i P x_i}{\sum_{i=1}^{n} w_i} \qquad y = \frac{\sum_{i=1}^{n} w_i P y_i}{\sum_{i=1}^{n} w_i}$$

Where Px_i and Py_i are the coordinates of the accident sites.

4.2 Incorporating True Distances Using the Weiszfeld Algorithm

In order to minimise the true distance we need to differentiate the expression for the total weighted distance. The derivative for the true distance is not defined at $(x_j, y_j) = (Px_i, Py_i)$ and there is no explicit formula for computing the optimal locations for the helicopter bases. Therefore we have to use an iterative procedure to solve this problem. In this project we have used a gradient steepest descent iterative method called the "Weiszfeld algorithm" in order to compute the new helicopter base locations [5].

4.3 Solution Procedure for Model 1

Figure 2 illustrates a schematic of the procedure for computing the optimal base locations for Pelikan 1, Pelikan 2 and AAD using Model 1 (Weighted Median). To find the optimal location for each base, we first use the formula derived by minimising the Squared-Euclidean distance to obtain a starting solution (Refer Section 4.1). This is then fed into the Weiszfeld Algorithm which finds the optimal base and returns it to the main loop where we update the subdivided regions by reallocating the accident sites to their nearest new bases. If there are any changes in the base locations, we resolve the problem using the new subdivided regions; otherwise the main loop stops execution.

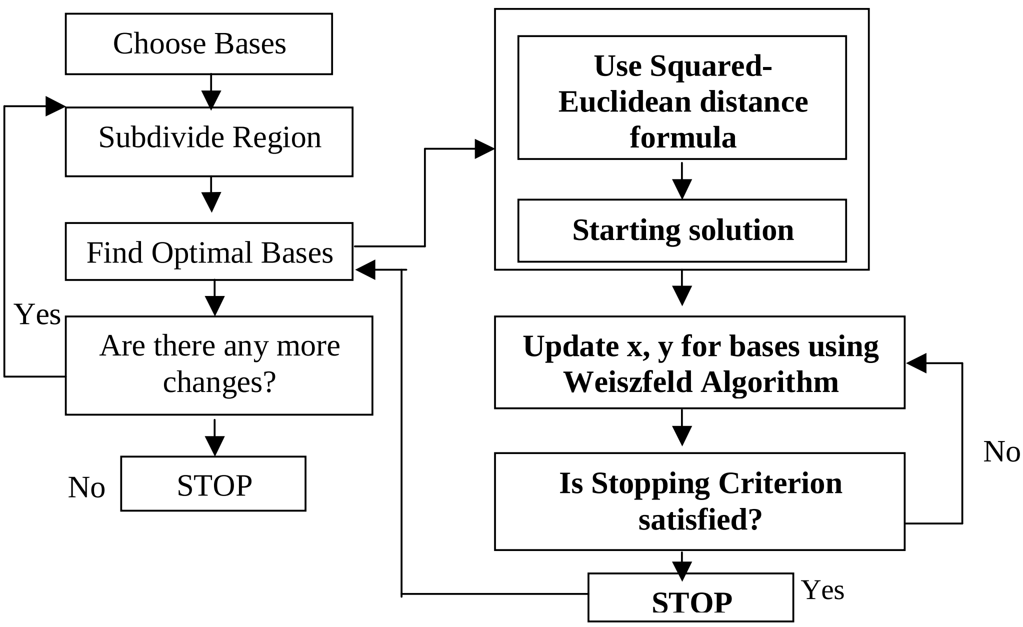

Figure 2: Procedure for finding optimal base locations using Model 1.

5 Solution Approach for Model 2

In order to solve Model 2, we applied some basic geometric properties of triangles. One of the properties states that if you move from side AB to the inside of the triangle ABC provided that the distance to the vertex A is the same as that to B, then you are moving along the perpendicular bisector of AB. The same applies for sides AC and BC. Therefore the point of intersection of any two bisectors of the triangle is equidistant from all three vertices of the triangle and the perpendicular bisector of the third side should also intersect at that point. The point of intersection is therefore the centre of the circumscribed circle or the circle that passes through the three vertices of the triangle centred at the intersection of any two bisectors [6]. For our problem, this centre point defines the optimal location of the helicopter base for three accident sites A, B, C.

5.1 Finding the Optimal Radius

The optimal radius is the smallest radius that encircles all other accident sites within a subdivided region and is centred at the intersection point of any two of the perpendicular bisectors of a triangle. However, if the triangle happens to be obtuse, then the point of intersection lies outside the circle. In this case we can take the midpoint of the largest side of the triangle (side opposite the obtuse angle) as being the centre of the circle. This gives rise to what we call the “two-point” search because two of the three vertices of the triangle lie on the circle and the third one lies inside the circle.

If the triangle in consideration happens to be acute, then the intersection of any two of the perpendicular bisectors of the triangle gives us the centre of the circle. This gives rise to the “three-point” search because, in this case, all the three points i.e. the three vertices of the triangle lie on the circle.

In order to calculate the radii, first the centre of the circle has to be found. The following sections describe the methods for finding the centre for the two-point and three-point search techniques.

5.2 Two-Point Search

If we apply the property of perpendicular bisectors to obtuse triangles that have two vertices lying on the circle, then the circle will be centred at the midpoint of the straight line through the two points as stated above. We can then examine how the radius of this circle can be used to search for possible optimal locations for the helicopter bases. The radius is only considered a possible optimal one if it encircles all the other accident sites in the region. If the distance between the centre of the circle and another accident site in the region is larger than the radius, it is simply ignored and a new set of points are chosen through which another circle is formed.

5.3 Three-Point Search

By reapplying the perpendicular bisector property (Section 5) for three points, where all three points lie on the circle, the intersection of the bisectors of the sides of the triangle gives us the centre of the circumscribed circle. This point is equidistant from the three vertices of the triangle. To ensure that the point of intersection falls inside the triangle defined by the three points, the angles of the triangle must be less than 90°. We then search for a possible optimal radius that encircles all the other accident sites in the region as we did for the two-point search technique.

5.4 Solution Procedure for Model 2

Figure 3 follows the procedure for computing the optimal base locations for Pelikan 1, Pelikan 2 and AAD using Model 2 (Un-weighted Centre). We use the two-point and three-point search heuristics (See Section 5.2 and 5.3) to find the smallest radii that can encircle all allocated sites in a subdivision. The circle formed by this radius is an optimal circle and the centre of this circle is the optimal location of the helicopter. These locations are returned to the main loop where we employ the same heuristic framework as we did for Model 1.

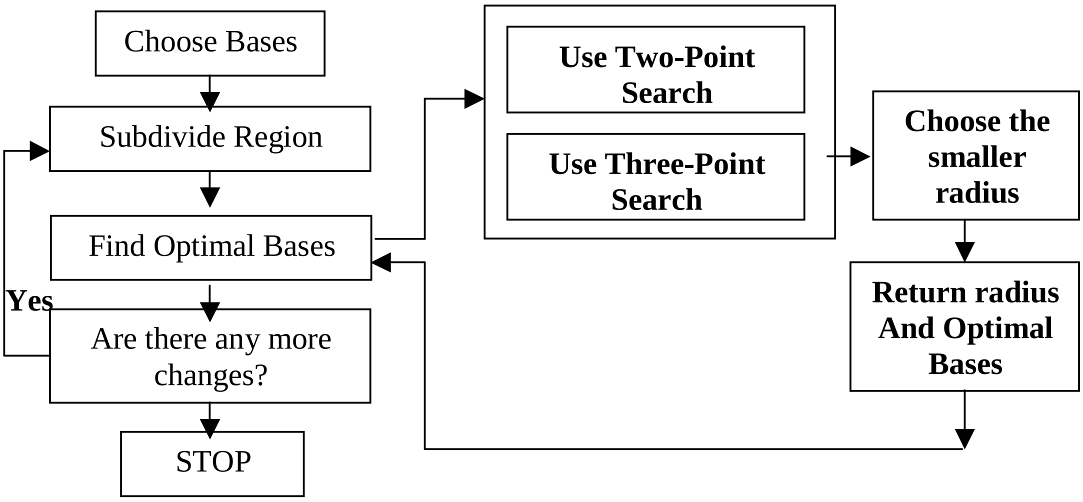

Figure 3: Procedure for finding optimal base locations using Model 2.

6 Results

The optimal location of each helicopter was subsequently found by implementing the heuristics described in previous sections. The resulting allocation of the accident sites to each of the newfound helicopter bases is shown in Figure 4 and 5.

6.1 Solution for Minimisation of the Total Weighted Distance

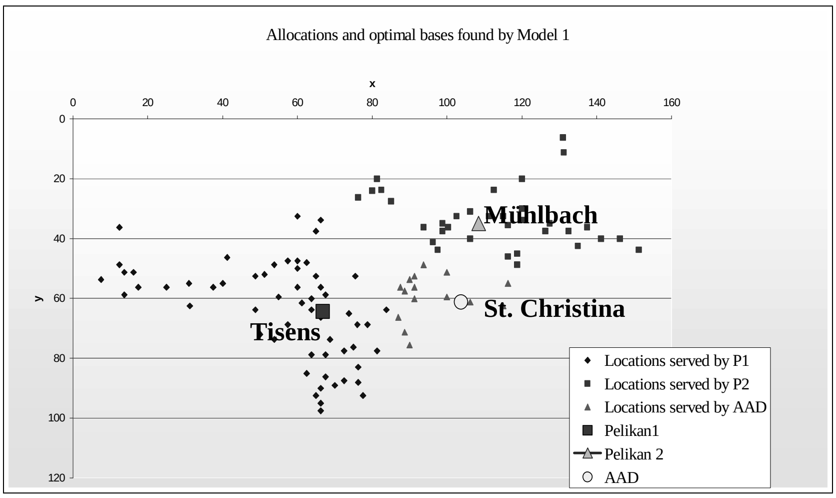

Figure 4: New Base locations found by Model 1.

The uneven allocations for Model 1, as depicted in Figure 4, are most likely to be the result of a locally optimal solution found by the applied heuristics. From Table 1 we can see that the number of sites (59) allocated to Pelikan 1 has not changed and is far greater than the number of accident sites allocated to Pelikan 2 and AAD which only have 34 and 16 allocated sites respectively.

Helicopter Locations	**x**	**y**	**Total Weighted Distance (km)**	**Maximum Un-weighted Distance (km)**	**Number of Missions**	**Number of Allocated Sites**
Pelikan1	66.83	64.32	19248.19	61.15	861	59
Pelikan2	108.47	34.93	10269.11	43.67	527	34
AAD	103.75	61.25	5448.54	19.80	540	16
			34965.85	**61.15**		

Table 1: Results for Model 1.

Compare with Original Values:

Helicopter Locations	**x**	**y**	**Total Weighted Distance (km)**	**Maximum Un-weighted Distance (km)**	**Number of Missions**	**Number of Allocated Sites**
Pelikan1	76	68.75	19620.22	71.33	892	59
Pelikan2	97.5	43.75	13856.29	53.75	603	39
AAD	98	60	4357.99	18.92	433	11
			37834.51	**71.33**		

Table 2: Original total weighted distance and maximum un-weighted distance.

Although Model 1 has not resulted in evenly allocated accident sites, it has improved the total weighted distance by 7% and the maximum un-weighted distance by 14%.

6.2 Solution for Minimisation of the Maximum Un-weighted Distance

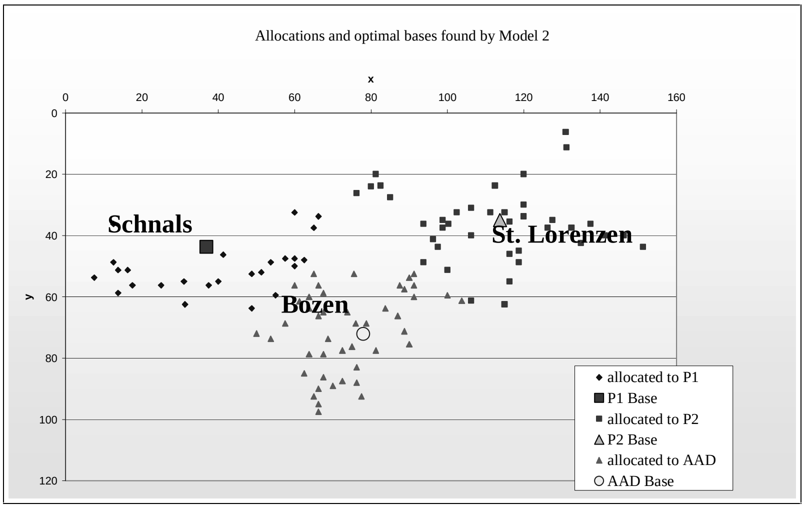

Figure 5: New Base locations found by Model 2.

Figure 5 shows that the allocated regions obtained by solving the Un-weighted Centre model are far better than those obtained by the Weighted Median Model because of the even distribution of accident sites to their respective helicopter bases (See Table 3).

Helicopter Locations	x	y	Total Weighted Distance (km)	Maximum Un-weighted Distance (km)	Number of Missions	Number of Allocated Sites
Pelikan1	36.87	43.75	7184.72	31.03	336	25
Pelikan2	113.75	35	16684.72	38.50	775	39
AAD	77.97	72.1	12267.72	27.97	817	45
			36137.16	**38.50**		

Table 3: Results for Model 2.

Results show an improvement in the maximum covered distance of about 46% whereas the total weighted distance has only improved by 4%. These results verify that Model 2 would be more suitable to use for finding the optimal base locations for the rescue helicopters.

6.3 Relating Model 1 with Model 2

We can explore the possibility of finding better solutions by studying the effect of moving the bases found by the Weighted Median Model in a straight line to the bases found by the Un-weighted Centre Model because the optimal locations found by the latter resulted in more evenly allocated regions. To do this, we moved the three bases simultaneously in a straight line through 100 steps and evaluated the maximum un-weighted distance and total weighted distance at each point. The resulting relation is shown in Figure 6.

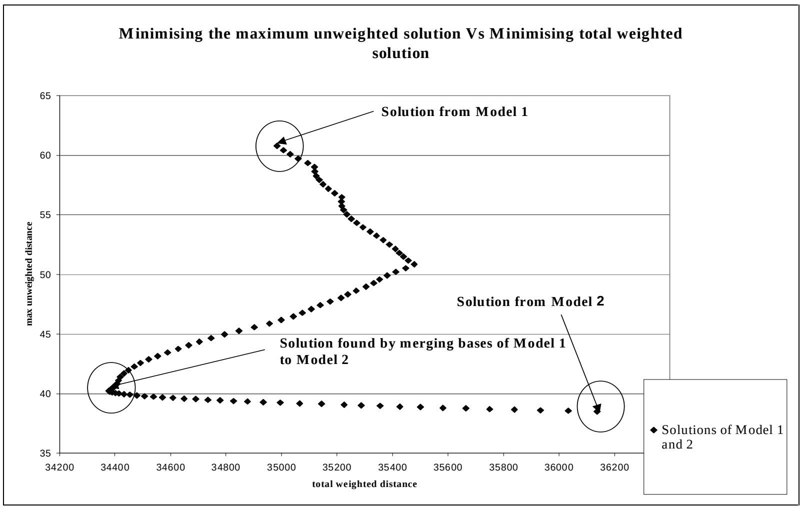

Figure 6: Objective of Model 1 versus Model 2

From Figure 6 we can infer that a better solution does exist but has not been found by the applied heuristics. It also confirms that the solution obtained by minimising the total weighted distance is not a globally optimal solution because the point (34386.285, 40.365) in the figure (Figure 6) gives a lower total weighted distance than the one found by implementing the Weiszfeld Algorithm in the heuristic for Model 1. This is certainly the best solution found so far. If the coordinates of the helicopter bases at this point (34386.285, 40.365) are evaluated, the optimal allocations can be obtained. Results in Table 4 show that the accident sites are more evenly allocated to the three helicopters.

Table 4 also shows an improvement in the total weighted distance of 9% and in the maximum un-weighted distance of 43%. The total weighted distance is lower than that found by solving Model 1 and Model 2 and the maximum un-weighted distance (40.36km) is not much worse than the one found by solving Model 2 (38.50km).

Helicopter Locations	x	y	Total Weighted Distance (km)	Maximum Un-weighted Distance (km)	Number of Missions	Number of Allocated Sites
Pelikan1	47.65	51.15	7776.35	40.24	401	35
Pelikan2	111.85	34.97	13496.61	40.36	660	37
AAD	87.25	68.19	13113.31	36.05	867	37
			34386.28	**40.36**		

Table 4: Optimal locations found by shifting bases to locations found in Model 2

In order to overcome these setbacks of solving the models separately we could in the future consider a bi-criteria approach with both the Weighted p-Median and the Un-weighted p-Centre objective functions. This will help find all possible efficient solutions for a combination of the two objectives from which we can establish a trade-off between the two models in order to decide which of the efficient solutions is most suitable for the Weißes Kreuz.

7 Conclusions

Our heuristic solution approach incorporated two basic models in order to solve the helicopter base location problem in South Tyrol. The first model was the Weighted Median Model which was used to improve the average response times and the second was the Un-weighted Centre Model to improve the worst response times. The first model gave us a local optimal solution for the helicopter locations which had uneven allocations of accident sites to the bases. The Un-weighted Centre Model, however, has shown an improvement of 46% by reducing the largest unweighted distance from 7l km to 38 km along with more evenly distributed accident sites to the helicopters as opposed to Model 1. By moving the optimal bases of Model 1 to those of Model 2 we were able to improve the average response times considerably to present a good solution to the Weißes Kreuz.

Acknowledgements

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