# Fuzzy Reasoning and Optimization Based on a Generalized Bayesian Network

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**Abstract**

Bayesian networks have been widely used as the knowledge bases with uncertainty. However, in most literatures, the uncertainty measure in Bayesian networks are limited in probability distributions and crisp variables, which restricts the practical usefulness of Bayesian networks when incomplete knowledge or linguistic vagueness is involved in the reasoning system. This study intends to develop a generalized Bayesian network in which the fuzzy variables, crisp variables, fuzzy parameters and crisp parameters can be considered. Based on the generalized Bayesian network, the fuzzy reasoning model for prediction, diagnosis, and optimization, can be designed. This study also develops the algorithms for fuzzy reasoning and optimization. The proposed network model will be applied to a two-echelon supply chain.
(*Generalized Bayesian Networks*; *Fuzzy Reasoning*; *Optimization*; *Supply Chain Management*)

## 1 Research Background and Objectives

Bayesian networks (Pearl 1988, Castillo et al. 1996, Castillo et al. 1997, Pearl 2000) are directed acyclic graphs (DAG) in which the nodes represent the variables, the arcs represent the direct causal influences between the linked variables, and the strengths of these influences are expressed by conditional probabilities. The semantics of Bayesian networks demands a clear correspondence between the topology of a DAG and the dependency relationships portrayed by it. They are widely used in knowledge representation and reasoning tools for various domains under uncertainty (Tatman and Shachter 1990, Dagum et al. 1992, Kao et al. 2000, Galán et al. 2002).

Several methods have been developed for solving abductive or diagnostic reasoning problems in Bayesian networks. Exact methods exploit the independence structure contained in the network to efficiently propagate uncertainty (Pearl 1988, Castillo et al. 1996, Castillo et al. 1997). Meanwhile, stochastic simulation methods provide an alternative approach suitable for highly connected networks, in which exact algorithms can be inefficient (Pearl 1988, Castillo et al. 1997). Recently, search-based approximate algorithms, which search for high probability configurations through a space of possible values, have emerged as a new alternative (Pool 1993). On the other hand, two key approaches have been proposed for symbolic inference in Bayesian networks, namely: the symbolic probabilistic inference algorithm (SPI) and symbolic calculations based on slight modifications of standard numerical propagation algorithms (Shacher et al. 1990, Castillo et al. 1996, Castillo et al. 1997).

The methods in the literatures have several limitations for reasoning from a Bayesian network:

1. All network nodes or domain variables must be crisp.
2. All parameters, including costs and utilities, of the network models are usually assumed crisp.
3. Different reasoning tasks, such as prediction, diagnosis and decision-making, cannot be done in a complete model.

This study intends to develop a generalized Bayesian network in which crisp nodes (variables), fuzzy nodes, and fuzzy parameters are included. Based on the generalized Bayesian network model, fuzzy reasoning techniques are designed to answer different queries from the network. The decision makers can make prognosis as well as diagnosis from the generalized Bayesian network with the fuzzy reasoning methods. Furthermore, alternative actions in response to the (potential) problems can be evaluated and selected based on the diagnostic report.

# 2 Problem and model development

This section introduces the generalized Bayesian network, the problem formulation, and the algorithm for fuzzy reasoning and optimization.

## 2.1 Generalized Bayesian networks

Generally, a Bayesian network is defined as (1).

$$BN = (V, L, P) \tag{1}$$

In (1), $V$ denotes the set of nodes (vertices), $L$ denotes the set of links (arcs), and $P$ denotes the probability model describing the network, where

$$L \subset V \times V \tag{2}$$

In most literatures, $V$, and $P$ are assumed crisp. If a crisp set is regarded as one special subset of fuzzy sets, then the definition of a Bayesian network can be extended into a generalized Bayesian network as follow.

$$GBN = (\widetilde{V}, \widetilde{L}, \widetilde{P}) \tag{3}$$

In (3), the set of nodes, probability distributions, and consequently the links, are no more limited to crisp sets, which allow greater modeling flexibilities of Bayesian networks. Furthermore, the composition of the node set $\widetilde{V}$ can be expressed as (4).

$$\widetilde{V} = \{\widetilde{V}_D, \widetilde{V}_R, \widetilde{V}_U\} \tag{4}$$

where $\widetilde{V}_D$ denotes the decision nodes, $\widetilde{V}_R$ represents the random nodes which is defined as the nodes in a conventional Bayesian network, $\widetilde{V}_U$ denotes the utility nodes which stand for the objectives to be optimized. By (4), an influence diagram is included in the definition.

## 2.2 Problem formulation

We first hypothesize a case of the two-echelon automotive supply chain (Naim et al. 2002).

**<u>Case 1</u>:**

After a field survey on the automotive supply chains, the engine assemblers and their suppliers can catch the whole picture of the supply chain performance. One main outputs of the field research is the cause-and-effect diagram shown in Figure 1 (Naim et al. 2002). In Figure 1, there are two levels of factors: the upper level of the customers (the engine assemblers, achromatic) and the lower level of the suppliers (in color). The arrows in the diagram represent the causal links between the keys of the two-echelon supply chain. The detailed description of Figure 1 is given in Table 1. Because there is a feedback loop in Figure 1, the two-echelon supply chain is a dynamic network.

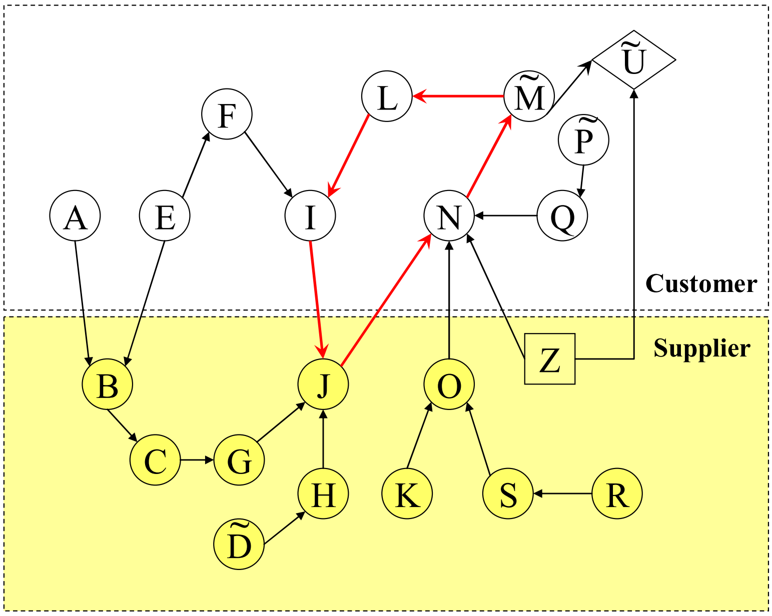


Figure 1. A generalized Bayesian network of the two-echelon supply chain

Let $V_D = \{Z\}, \widetilde{V}_R = \{X, \widetilde{Y}\}, \widetilde{V}_U = \{\widetilde{U}\}$, where $X$ stands for the crisp random node set and $\widetilde{Y}$ stands for the fuzzy random node set. For the crisp nodes, we use the uppercase letters to represent the variables and lowercase letters for their associated values. For example, $C^t \in \{0,1\}$ represents the dichotomy between low risk of obsolescence and high risk of obsolescence at time *t*, $+c^t$ stands for $C^t = 1$ and $\neg c^t$ stands for $C^t = 0$. On the other hand, we assign a triangular membership function to represent a fuzzy node. Every fuzzy node $\widetilde{Y}$ is denoted by $(\underline{y}, y^*, \overline{y})$, where $\underline{y}$ and $\overline{y}$ represent the lower and upper limit of the support of fuzzy set $\widetilde{Y}$, and $y^*$ stands for the most confident value of $\widetilde{Y}$. That is, $\mu_{\widetilde{Y}}(y^*) = 1$ and $\mu_{\widetilde{Y}}(y) = 0$ for $y \geq \overline{y}, y \leq \underline{y}$. Assume that all the probability and possibility distributions have been learned and given in Table 2 and Table 3. Remarkably, the probability distributions of some crisp nodes conditioned on the manifestation of their fuzzy parents, such as *H*, *L*, and *Q*. To cope with these cases, we partition the support of the fuzzy continuous parents into limited sub-domains, and then approximate the crisp children's conditional probability on the sub-domains (Miller and Rice 1983, Keefer 1994).

Table 1. The states and description of the nodes in Figure 1

| Node | Level | Description | State |
|---|---|---|---|
| *Crisp nodes* | | | |
| $A$ | Customer | Product variety | $+a$: *high*, $\neg a$: *low* |
| $B$ | Supplier | Product range | $+b$: *high*, $\neg b$: *low* |
| $C$ | Supplier | Risk of obsolescence | $+c$: *high*, $\neg c$: *low* |
| $E$ | Customer | Design specification alterations | $+e$: *high*, $\neg e$: *low* |
| $F$ | Customer | B.O.M accuracy | $+f$: *high*, $\neg f$: *low* |
| $G$ | Supplier | Finished goods safety stock | $+g$: *high*, $\neg g$: *low* |
| $H$ | Supplier | Lack of raw materials at use | $+h$: *high*, $\neg h$: *low* |
| $I$ | Customer | Schedule alterations on suppliers | $+i$: *high*, $\neg i$: *low* |
| $J$ | Supplier | Schedule adherence | $+j$: *high*, $\neg j$: *low* |
| $K$ | Supplier | Production capacity constraint | $+k$: *high*, $\neg k$: *low* |
| $L$ | Customer | Schedule build alterations | $+l$: *high*, $\neg l$: *low* |
| $N$ | Customer | Lack of components at use | $+n$: *high*, $\neg n$: *low* |
| $O$ | Supplier | Scheduling flexibility | $+o$: *high*, $\neg o$: *low* |
| $Q$ | Customer | Lost stock at use | $+q$: *high*, $\neg q$: *low* |
| $R$ | Supplier | Set-up times and costs | $+r$: *high*, $\neg r$: *low* |
| $S$ | Supplier | Volume of batch production | $+s$: *high*, $\neg s$: *low* |
| $Z$ | Supplier | Actions to treat component lack *(decision node)* | $z_1$: *strategic outsourcing*<br>$z_2$: *expansion by old tech*<br>$z_3$: *expansion by new tech* |
| *Fuzzy nodes* | | | |
| $\tilde{D}$ | Supplier | Stock control performance | (*low, most possible, high*) |
| $\tilde{M}$ | Customer | Build capability | (*low, most possible, high*) |
| $\tilde{P}$ | Customer | Stock control performance | (*low, most possible, high*) |
| $\tilde{U}$ | Customer | Value of order fulfillment *(utility node)* | (*low, most possible, high*) |

As a supplier of the leading engine assembler Company C, Company S monitors the key indicators from the diagram periodically. The evidence collected are poor schedule adherence ($J^t = 0$), considerable schedule alterations ($I^t = 1$), large product range ($B^t = 1$), high risk of obsolescence ($C^t = 1$), limited finished goods ($G^t = 0$), large production capacity constraint ($K^t = 1$), poor scheduling flexibility ($O^t = 0$), large set-up times/costs ($R^t = 1$) and had large batch production ($S^t = 1$).

Table 2. The distributions for the nodes in Figure 1

| | |
|---|---|
| *Crisp random nodes* | |
| $P(+a^t) = 0.70$ | |
| $P(+b^t \mid +a^t, +e^t) = 0.90$ | $P(+b^t \mid \neg a^t, +e^t) = 0.60$ |
| $P(+b^t \mid +a^t, \neg e^t) = 0.80$ | $P(+b^t \mid \neg a^t, \neg e^t) = 0.20$ |
| $P(+c^t \mid +b^t) = 0.85$ | $P(+c^t \mid \neg b^t) = 0.20$ |
| $P(+e^t) = 0.40$ | |
| $P(+f^t \mid +e^t) = 0.15$ | $P(+f^t \mid \neg e^t) = 0.90$ |
| $P(+g^t \mid +c^t) = 0.10$ | $P(+g^t \mid \neg c^t) = 0.80$ |
| $P(+h^t \mid d^t_{>0.6}) = 0.05$ | $P(+h^t \mid d^t_{\le 0.6}) = 0.90$ |
| $P(+i^t \mid +f^t, +l^t) = 0.80$ | $P(+i^t \mid \neg f^t, +l^t) = 1.00$ |
| $P(+i^t \mid +f^t, \neg l^t) = 0.01$ | $P(+i^t \mid \neg f^t, \neg l^t) = 0.50$ |
| $P(+j^t \mid +g^t, +h^t, +i^t) = 0.20$ | $P(+j^t \mid +g^t, \neg h^t, +i^t) = 0.50$ |
| $P(+j^t \mid +g^t, +h^t, \neg i^t) = 0.60$ | $P(+j^t \mid +g^t, \neg h^t, \neg i^t) = 0.99$ |
| $P(+j^t \mid \neg g^t, +h^t, +i^t) = 0.00$ | $P(+j^t \mid \neg g^t, \neg h^t, +i^t) = 0.50$ |
| $P(+j^t \mid \neg g^t, +h^t, \neg i^t) = 0.60$ | $P(+j^t \mid \neg g^t, \neg h^t, \neg i^t) = 0.80$ |
| $P(+k^t) = 0.50$ | |
| $P(+l^t \mid m^t_{\ge 0.9}) = 0.10$ | $P(+l^t \mid m^t_{\le 0.8}) = 0.90$ |
| $P(+n^t \mid +j^{t-1}) = 0.10$ | $P(+n^t \mid \neg j^{t-1}) = 0.50$ |
| $P(+o^t \mid +k^t, +s^t) = 0.00$ | $P(+o^t \mid \neg k^t, +s^t) = 0.70$ |
| $P(+o^t \mid +k^t, \neg s^t) = 0.60$ | $P(+o^t \mid \neg k^t, \neg s^t) = 0.95$ |
| $P(+q^t \mid p^t_{>0.6}) = 0.10$ | $P(+q^t \mid p^t_{\le 0.6}) = 0.50$ |
| $P(+r^t) = 0.50$ | |
| $P(+s^t \mid +r^t) = 0.70$ | $P(+s^t \mid \neg r^t) = 0.30$ |
| *Fuzzy random nodes* | |
| $Pos(\widetilde{d}^t) = (0.30, 0.60, 0.90)$ | |
| $Pos(\widetilde{m}^t \mid +n^t) = (0.50, 0.60, 0.80)$ | $Pos(m^t \mid \neg n^t) = (0.90, 0.95, 1.00)$ |
| $Pos(\widetilde{p}^t) = (0.50, 0.60, 0.70)$ | |

Table 3. The distributions conditioned on decision nodes

| | |
|---|---|
| $Z = z_1$ | |
| $P(+n^t \mid +o^t, +q^t, z_1) = 0.20$ | $P(+n^t \mid \neg o^t, +q^t, z_1) = 0.60$ |
| $P(+n^t \mid +o^t, -q^t, z_1) = 0.01$ | $P(+n^t \mid \neg o^t, -q^t, z_1) = 0.10$ |
| $Z = z_2$ | |
| $P(+n^t \mid +o^t, +q^t, z_2) = 0.10$ | $P(+n^t \mid \neg o^t, +q^t, z_2) = 0.30$ |
| $P(+n^t \mid +o^t, -q^t, z_2) = 0.00$ | $P(+n^t \mid \neg o^t, -q^t, z_2) = 0.10$ |
| $Z = z_3$ | |
| $P(+n^t \mid +o^t, +q^t, z_3) = 0.05$ | $P(+n^t \mid \neg o^t, +q^t, z_3) = 0.10$ |
| $P(+n^t \mid +o^t, -q^t, z_3) = 0.00$ | $P(+n^t \mid \neg o^t, -q^t, z_3) = 0.05$ |

There is one decision node $Z$ and one utility node $\widetilde{U}$. The decision node represents the solution set which company S may choose to treat the poor schedule adherence. There are three alternatives in the solution set, that is $Z = \{z_1, z_2, z_3\}$, where $z_1$ is to develop the strategic outsourcing alliance, $z_2$ is to expand the manufacturing capacity with current technologies, and $z_3$ is to expand the capacity with new manufacturing technologies. The estimated costs of the three decision alternations are 500, 1,000 and 2,000 thousand dollars, respectively. The utility $\widetilde{U}^t = f(z, \widetilde{m}^t)$ is determined by $\widetilde{M}$ (Build capability), $Z$ (the decision), which is converted into money amount. The objective is to maximize the net utility.

### 2.3 Decomposing the loop

Now Company S needs to compute the posterior distributions of every proposition in the system backward for $n$ periods, given the evidence set $\check{\mathbf{E}}$ ={ě}= $\{B^t$=1, $C$=1, $G^t$=0, $I^t$=1, $J^t$=0, $K^t$=1, $O^t$=0, $R^t$=1, $S^t$=1$|1 \le t \le n\}$.

The joint distribution of the network, $L(x, \widetilde{y})$, from time $t$ = 1 to $n$ is as (5).

$$\begin{aligned}
&L(x,\widetilde{y}) = P(x \mid z) \bullet Pos(\widetilde{y}) \\
&= P(a^1, a^2, ..., a^n, b^1, b^2, ..., b^n, ..., s^1, s^2, ..., s^n \mid z) \otimes Pos(\widetilde{d}, \widetilde{m}, \widetilde{p}) \\
&= P(a^1)P(b^1 \mid a^1, e^1)P(c^1 \mid b^1)P(e^1)P(f^1 \mid e^1)P(g^1 \mid c^1)P(h^1 \mid \widetilde{d}^1) \\
&\quad \times P(i^1 \mid f^1, l^1)P(j^1 \mid i^1, g^1, h^1)P(k^1)P(l^1 \mid \widetilde{m}^1)P(n^1 \mid o^1, q^1, z) \\
&\quad \times P(o^1 \mid s^1, k^1)P(q^1 \mid \widetilde{p}^1)P(r^1)P(s^1 \mid r^1) \\
&\quad \otimes [Pos(\widetilde{d}^1) \wedge Pos(\widetilde{m}^1 \mid n^1) \wedge Pos(\widetilde{p}^1)] \qquad (5) \\
&\quad \times \prod_{t=2}^{n} [P(a^t)P(b^t \mid a^t, e^t)P(c^t \mid b^t)P(e^t)P(f^t \mid e^t)P(g^t \mid c^t)P(h^t \mid \widetilde{d}^t) \\
&\quad \times P(i^t \mid f^t, l^t)P(j^t \mid i^t, g^t, h^t)P(k^t)P(l^t \mid \widetilde{m}^t)P(n^t \mid j^{t-1}, o^t, q^t, z) \\
&\quad \times P(o^t \mid s^t, k^t)P(q^t \mid \widetilde{p}^t)P(r^t)P(s^t \mid r^t)] \\
&\quad \bullet [Pos(\widetilde{d}^t) \wedge Pos(\widetilde{m}^t \mid n^t) \wedge Pos(\widetilde{p}^t)].
\end{aligned}$$

In Figure 1, a feedback loops exists among *I*, *J*, *N*, $\widetilde{M}$ and *L*. If we take a time expansion aspect, Figure 1 can be expended into Figure 2. The equation in (5) involves both joint probability and possibility functions, so we use "•" to denote the product operator linking the crisp and fuzzy parameters.

The term $P(n^t \mid j^{t-1}, o^t, q^t, z)$ embraces contemporaneous dependencies at time $t$ and non-contemporaneous dependencies at $t-1$. There are two simple parameterized decompositions used commonly by time-series analysts as in the following remark.

**Remark 1**: Additive and multiplicative decomposition

Let $\omega$ denotes the likelihood that $n^t$ predicted from the information at period $t$, and $(1-\omega)$ denotes the likelihood that $n^t$ predicted from the information prior to time $t$. In the additive decomposition, the conditional probability function $P(n^t \mid j^{t-1}, o^t, q^t, z)$ can be given by (6).

$$P(n^t \mid j^{t-1}, o^t, q^t) = \omega P(n^t \mid o^t, q^t, z) + (1-\omega) P(n^t \mid j^{t-1}) \qquad (6)$$

In the multiplicative decomposition, the conditional probability function is as follow.

$$P(n^t \mid j^{t-1}, o^t, q^t) = \gamma\, P(n^t \mid o^t, q^t, z)^{\omega} \times P(n^t \mid j^{t-1})^{1-\omega} \tag{7}$$

where $\gamma$ is a constant that normalizes the probability distributions to unify. Considering the properties of Case 1, this study will use additive decomposition in (6).

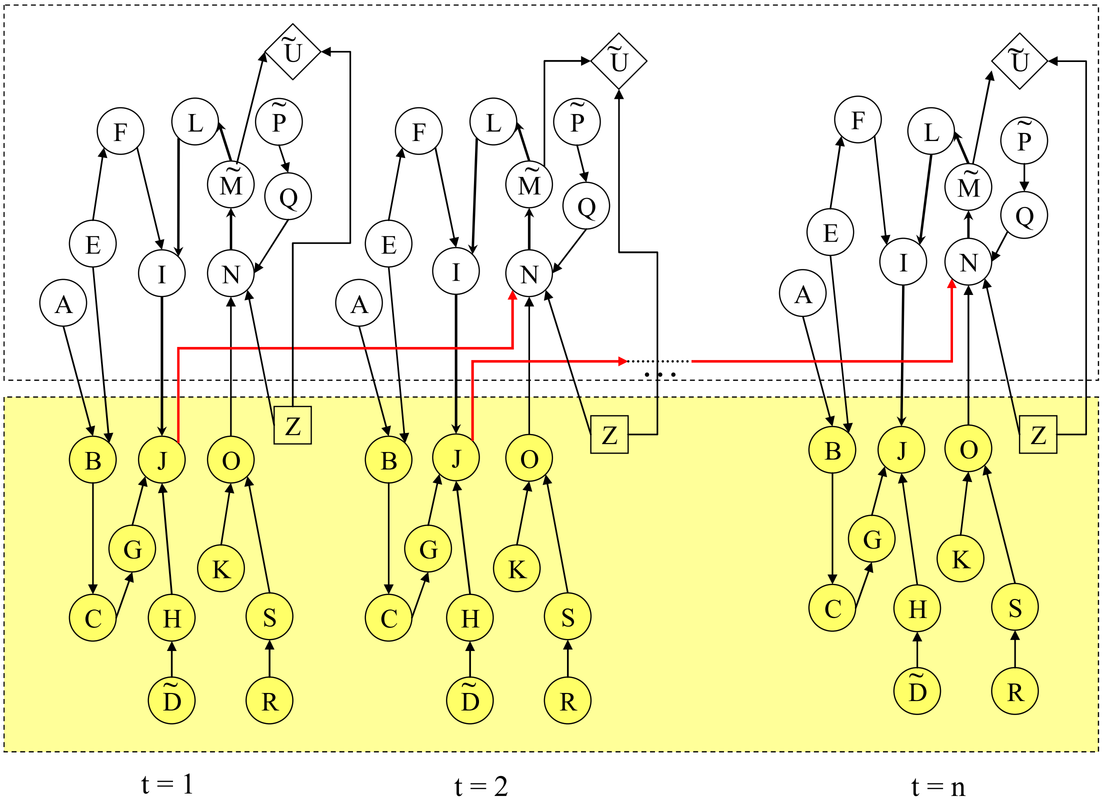


Figure 2. Time expansion of the dynamic network in Figure 1

## 2.4 Distributions of the nodes

For a binary node *X*, we denote by $w_X$ the states of all variables which *X* is conditioned on, including *X*'s parents, *X*'s children, and the other direct parents of *X*'s children. The value of *X* will be chosen by tossing a coin that favors 1 over 0 by a ratio of $P(+x \mid w_X)$ to $P(\neg x \mid w_X)$. The distribution of each crisp variable *X* conditioned on the values $w_X$ of all other variables in the system, $P(x \mid w_X)$, can be calculate by local computations (Pearl 1988). Similarly, the possibility of $\widetilde{Y}$ conditioned on $w_{\widetilde{Y}}$, $Pos(\widetilde{y} \mid w_{\widetilde{Y}})$, is determined by the states of $\widetilde{Y}$'s parents, $\widetilde{Y}$'s children, the other direct parents of $\widetilde{Y}$'s children.

For the fuzzy nodes, $Pos(\widetilde{y} \mid w_{\widetilde{Y}})$ at time $t$ are list as (8)-(11).

$$Pos(\widetilde{d}^{\,t} \mid w_{\widetilde{D}^t}) = \alpha Pos(\widetilde{d}^{\,t}) \bullet P(h^t \mid \widetilde{d}^{\,t}) \tag{8}$$

$$Pos(\widetilde{m}^t \mid w_{\widetilde{M}^t}) = \alpha Pos(\widetilde{m}^t \mid n^t) \bullet P(l^t \mid \widetilde{m}^t) \tag{9}$$

$$Pos(\widetilde{p}^t \mid w_{\widetilde{P}^t}) = \alpha Pos(\widetilde{p}^t) \bullet P(q^t \mid \widetilde{p}^t) \tag{10}$$

$$\widetilde{u} = f(z, \widetilde{m}) = \psi(z) \cdot \widetilde{m} \tag{11}$$

where $\alpha$ is the normalizing constant and $\psi(z)$ is the base of added value from the decision $z$. In the fuzzy simulation, the operator "$\bullet$" is replaced with "$\wedge$".

### 2.5 Algorithm for fuzzy reasoning and optimization

There are at least three tasks are possibly performed in the generalized Bayesian networks: prediction, diagnosis, and decision-making. This study proposes a fuzzy simulation technique for solving the reasoning problems on the GBN.

**Procedure *Simulation_1***
  Initialize $i = 0$.
  Read **CrispSet**, **FuzzySet**, **EvidenceSet**, **UnknownNodeSet**.
  Repeat while not end of **UnknownNodeSet**:
    Read *UnknownNode* from **UnknownNodeSet**.
    IF *UnknownNode* $\in$ **CrispSet** THEN
      $X \leftarrow$ *UnknownNode*.
      Run ***StochasticSimulation***.
    ELSE
      $Y \leftarrow$ *UnknownNode*.
      Run ***FuzzySimulation***.
  End of Repeat.
End. /* ***End of Procedure Simulation_1*** */

**Procedure *FuzzySimulation*** /* for fuzzy unknown nodes */
  Set $\lambda$. /* the λ-level cut of fuzzy sets */
  Set $Pos(y \mid w_Y)$. /* as in (8)-(11) */
  $y\,[i] \leftarrow$ SAMPLE($Pos(y_\lambda \mid w_{\tilde{Y}})$).
  $i = i+1$.
End. /* ***End of Procedure FuzzySimulation*** */

**Procedure *StochasticSimulation*** /* for crisp unknown nodes */
  $r \leftarrow P(X=1 \mid w_X) / P(X=0 \mid w_X)$.
  IF RANDOM( ) $\leq r/(r+1)$ THEN
    $X \leftarrow 1$, $COUNT\_X1 = COUNT\_X1 + 1$.
  ELSE
    $X \leftarrow 0$, $COUNT\_X0 = COUNT\_X0 + 1$.
  $BEL(X) \leftarrow COUNT\_X1 / (COUNT\_X1 + COUNT\_X0)$.
End. /* ***End of Procedure StochasticSimulation*** */

## 3 Results

This study sets the $\lambda$-cut of the fuzzy sets at 0.5 and 0.9. For every decision alternative ($z_1, z_2, z_3$), the simulation program was executed for 1000 iterations. The results show the distributions of the utilities associated with different strategies and different λ-level at every time period. The detailed output at $\lambda$-level = 0.5 is listed in Table 4.

Table 4. The distributions of utility ($\lambda$-level = 0.5)

| | | $t = 1$ | $t = 2$ | $t = 3$ |
|---|---|---|---|---|
| $z_1$ | $BEL(\widetilde{m}^t \mid \breve{e}, z_1)$ | (0.5450,0.6592, 0.9774) | (0.5451, 0.6393, 0.9765) | (0.5451, 0.6429, 0.9773) |
| | *Gross utility* | (545,659,977) | (545,639,976) | (545,642,977) |
| | *Net utility* | (45,159,477) | (45,139,476) | (45,142,477) |
| $z_2$ | $BEL(\widetilde{m}^t \mid \breve{e}, z_2)$ | (0.5450, 0.6857, 0.9772) | (0.5451, 0.6529, 0.9775) | (0.5452, 0.6446, 0.9770) |
| | *Gross utility* | (1090,1372,1954) | (1090,1306,1954) | (1090,1290,1954) |
| | *Net utility* | **(90,372,954)** | **(90,306,954)** | **(90,290,954)** |
| $z_3$ | $BEL(\widetilde{m}^t \mid \breve{e}, z_3)$ | (0.5452, 0.7418, 0.9771) | (0.5452, 0.6724, 0.9774) | (0.5450, 0.6706, 0.9770) |
| | *Gross utility* | (1635,2226,2931) | (1635,2016,2931) | (1635,2013,2931) |
| | *Net utility* | (-365,226,931) | (-365,16,931) | (-365,13,931) |

Table 5. The results of diagnosis and prediction ($\lambda$-level = 0.5)

| | $t$ =1 | $t$ =2 | $t$ =3 |
|---|---|---|---|
| *Crisp nodes* | | | |
| $BEL(+a^t \mid \breve{e})$ | **0.8240** | **0.8400** | **0.8330** |
| $BEL(+e^t \mid \breve{e})$ | 0.5930 | 0.5380 | 0.5470 |
| $BEL(+f^t \mid \breve{e})$ | 0.4060 | 0.4500 | 0.4440 |
| $BEL(+h^t \mid \breve{e})$ | 0.6100 | 0.5460 | 0.6720 |
| $BEL(+l^t \mid \breve{e})$ | **0.8570** | **0.8850** | **0.8810** |
| $BEL(+n^t \mid \breve{e})$ | **0.7760** | **0.8300** | **0.8200** |
| $BEL(+q^t \mid \breve{e})$ | 0.6680 | 0.4710 | 0.5050 |
| *Fuzzy nodes* | | | |
| $BEL(\widetilde{d}^t \mid \breve{e})$ | (0.4353, 0.5815, 0.7568) | (0.4352, 0.5903, 0.7570) | (0.4353, 0.5694, 0.7575) |
| $BEL(\widetilde{p}^t \mid \breve{e})$ | (0.5251, 0.6592, 0.9774) | (0.5252, 0.6393, 0.9765) | (0.5451, 0.6429, 0.9773) |

The utilities are expressed in triangular distributions. Based on the results, $z_3$ results in the maximal gross utilities among all decision alternatives. However, $z_2$ outperforms the other two alternatives in net utility. Hence, the optimal decision for the supplier is $z_2$ (capacity expansion based on current technologies). The belief distributions of the other unknown random nodes at $\lambda$-level = 0.5 are listed in Table 5.

## 4 Conclusions

This study extends the traditional Bayesian networks into generalized Bayesian networks, in which different uncertainty measures (probability and possibility distributions) and different nodes (decision, random and utility nodes) can be considered. Different reasoning tasks, such as prediction, diagnosis, and optimization, can be finished in the network model. This work also develops the fuzzy simulation algorithms for answer queries on the generalized Bayesian networks. The author expects to make some contributions to reasoning from various systems, such as supply chain management, medical informatics, and so on.

## 5 References


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