Multi-Criteria Methods for Unit Crewing in the Airline Tour-of-Duty Planning Problem

Bassy Tam
Department of Engineering Science
University of Auckland
New Zealand
mtam017@ec.auckland.ac.nz

Abstract
In crew scheduling, the aim of unit crewing is to keep crew members from different crew ranks together to operate a sequence of flights as a unit as much as possible. When the crew members of different ranks operate together for a sequence of flights, the sequence is unit crewed. A unit crewed solution is considered to be more robust in the sense that aircraft departures are less likely to be delayed due to waiting for a member of a crew.

It is usual to solve the ToD planning problem for each crew rank as a separate problem. To maximize unit crewing in the Tour-of-Duty (ToD) solution, a set of ToDs for one rank in terms of minimal cost is constructed and a weighted sum method is used to penalize any tour for the other rank that does not contain the same flight sequence.

We believe that a model that solves the two ToD planning problems simultaneously can improve the number of unit crewing sequences without increasing the total crew cost. We use the number of unit crewed flight as a second objective and a branch and bound strategy to ensure that as many unit crewing sequences can be selected as possible.

1 Airline Crew Scheduling
Commercial airlines are required to solve many resource scheduling problems to ensure that aircraft and aircrew are available for each scheduled flight. The aircrew scheduling problem consists of two distinct sub-problems, the tour-of-duty (ToD) planning problem and the rostering problem.

The ToD planning problem consists of finding sequences of flights to partition the flight schedule into ToDs that can be flown by one crew member. The rostering process involves the allocation of the planned ToDs from the first ToD planning problem to individual crew members to form a line-of-work (LoW) for each crew member over the rostering period.

A ToD is an alternating sequence of duty periods and layovers, where duty periods comprise one or more flights, and may also include passengering flights. A passengering flight is one on which a crew member travels as a passenger in order to be positioned at a particular airport for a subsequent operating flight or to return to their home base.

The first duty period of a ToD must start at a crew base, and the last duty period must end at the same crew base. An airline might have several bases (cities) at which crew are domiciled. Each ToD will have an associated crew complement made up of a number of crew of some ranks.
A set partitioning model provides an underlying mathematical model for the ToD planning and the rostering sub-problems of the aircrew scheduling problem. The set partitioning problem (SPP) is a specially structured zero-one integer linear programme with the form:

\[
\begin{align*}
\text{Minimise} & \quad z = c^T x \\
\text{subject to} & \quad Ax = e \\
& \quad x \in \{0, 1\}^n
\end{align*}
\]  

(SPP)

where \( e = (1, 1, \ldots, 1)^T \) and \( A \) is a matrix of zeros and ones.

In the basic ToD planning model, each column or variable in the SPP corresponds to one feasible ToD that can be flown by some crew member. Each constraint in the SPP corresponds to a particular flight and ensures that each flight is included in exactly one ToD. The elements of the \( A \) matrix can then be defined as

\[
a_{ij} = \begin{cases} 
1, & \text{if the } j^{th} \text{ ToD includes the } i^{th} \text{ flight as operating flight,} \\
0, & \text{otherwise.}
\end{cases}
\]

The ToD planning model is usually augmented with additional constraints that permit restrictions to be imposed on the number of crew resources included from each crew base; this set of constraints is referred to as \textit{crew base balancing constraints} and denoted by \( Mx \ \{\leq, =, \geq\} \ b \). Because these crew base balancing constraints typically have non-unit right-hand-side values, the ToD planning model is describe as a generalised set partitioning problem (GSSP).

Optimisation systems for ToD planning often incorporate dynamic (delayed) column generation based on resource-constrained shortest path methods. A \textit{dynamic column generation} technique generates columns during the optimization process. In this way, the size of the problem increases dynamically, but most of the feasible ToDs will never be generated.

The integer programming problems are solved by a branch and price algorithm which uses constraint branching. This technique selects pairs of sectors which could be flown consecutively (\textit{follow-on} flight pair) in a ToD to bound on (see Butchers et al.).

\section{Operational Robustness}

The ToD planning problem is solved well before the flight schedule becomes operational. In this planning stage, all flights are assumed to have departure times that are both fixed and known. This assumption is often proven wrong when the crew schedule is actually implemented.

When crew members’ schedules are disrupted in operations, they are nonetheless guaranteed to be paid for their original scheduled workload. In addition, if delays increase their flying or ground time, they may be entitled to additional compensation. Furthermore, disruptions may require the use of reserve crews to get back on schedule and originally scheduled crew might not be able to continue on their duty because of violating rules. Clearly, the cost associated with implementing a ToD solution may vary significantly from the planned cost.

So airlines do not only require minimum cost solutions, but are also very interested in robust solutions. A robust ToD planning problem is the problem of obtaining aircrew schedules in planning that are not necessarily optimal in terms of the crew cost in plan but that yield low crew cost in operations.

Approaches to robust aircrew scheduling have been developed only recently, but all of the following approaches have different measure of operational robustness.
**Expected ToD Cost by Simulation**

Schaefer et al., 2001 solve a problem very similar to the original ToD planning model. However, they replace the objective coefficients of the $j^{th}$ ToD in the model with the expected cost of the $j^{th}$ ToD. The observed effect of this approach is to produce ToDs with lower operational cost than those produced with planned cost.

The difficult aspect of this problem is in computing the cost coefficients, given that the expected cost of a ToD depends in part on the other ToDs in the crew schedule. For every ToDs they compute the excepted cost by running simulations under the assumption that the excepted cost is independent of the other ToDs in a crew schedule.

**Maximizing Robustness using Bi-Criteria Optimization**

Ehrgott and Ryan, 2002 illustrate that “a robust solution is one in which crew changing aircraft is discouraged if insufficient ground time occurs to compensate late arrivals”. In other words, a robust solution would have the property that if an upstream flight is likely to be delayed, crew should not be scheduled to change aircraft for a successive flight, which leaves after only minimal ground time.

Thus, crews change aircraft between operating flight sectors less frequently in a robust ToD solution. Based on this illustration, they develop an objective function to penalize ToDs which are not robust. They then try to minimize this objective while at the same time maintaining a cost effective solution.

There is a trade-off between minimizing the crew cost and minimizing the non-robustness penalty. A schedule that minimizes the non-robustness measures will have high crew cost. To circumvent this problem, Ehrgott and Ryan used the elastic constraint method to solve the LP relaxation on their restricted master ToD planning model.

The traditional ToD planning problem is solved first, thus giving a planned crew cost. Then the non-robustness objective is minimized with an added constraint to control the crew cost – i.e. to specify an upper bound, so that the planned crew cost objective is not too far from the minimum crew cost objective. They then use an elastic constraint that allows a small violation of the cost constraint if robustness can be improved in the branch and bound process.

**Maximizing Robustness using Stochastic Programming**

Yen and Birge, 2000 solve the robust ToD planning model with a similar robustness measure as described above (Ehrgott and Ryan) as a two-stage stochastic binary programming model with recourse by assuming the connection time is a random variable.

Given a crew schedule, the recourse problem is a large-scale LP to measure the cost of delays, with the first stage problem being the traditional ToD planning with GSPP formulation. They develop a method based on follow-on branching to solve the model.

They sample 100 disruption scenarios and evaluate the solution of the second stage LP for each scenario to determine the “switching cost” associated with the aircraft changes. The “switching cost” is then passed back to the first stage problem to remove any “expensive” aircraft changes, by branching on the sector pair with the highest switching cost.

The main drawback for this approach is that it is very computationally expensive, and they only show computational results on a problem with 50 flight segments, which is rather small.
Chebalov and Klabjan, 2002 solve the ToD planning problem based on the observation of a recovery procedure that uses crew swaps. When a crew member is delayed or has reached a limit on his/her flying time for a duty or ToD, it would be highly desirable to have an alternative crew member available with whom it could swap one or more flights.

In addition to the traditional objective of minimizing the ToD cost, they introduce a new objective of maximizing the number of opportunities for crew swapping. Thus, their model is a bi-criteria optimization model.

3 Unit Crewing

In some airlines, crew members may have different employment contracts, different scheduling rules and different pay schemes, even if they belong to the same crew type. So ToD planning problems have to be solved separately for each crew rank. This might cause a crew to split up so that some members of the crew may join other crew members to make up a crew on one flight and some crew members may join a second crew for another flight. This splitting of the crew is not robust from an operational point of view.

If an incoming flight arrives late, the crew members on that flight will arrive late which causes the crew members on board to be late for the downstream flight if the connection time between the two flights is tight. If members of the crew in the upstream flight split up and join other crew members to form new crews for downstream flights, then more than one outgoing flight will be affected.

In fact, this affects more seriously the technical crew, as there are fewer options for the recovery procedure for technical crew than for the flight attendants, since the number of reserve flight attendants is higher than the number of reserve pilots, and flight attendants are able to work on different types of aircraft, whereas pilots can only operate certain fleet type(s).

Normally, there are two pilots (the Captain and the First Officer) to operate one aircraft in Air New Zealand domestic operations. Figure 1 shows a flight $F_r$ that has one
Captain ($F^C_r$) and one First Officer ($F^F_r$) scheduled on its operation, with flight $F_s$ and flight $F_t$ both having short connection time to $F_r$. $F^C_r$ was scheduled to join another First Officer to operate $F_s$ and $F^F_r$ was scheduled to join another Captain to operate $F_t$. If $F_r$ is late on its arrival, then $F^C_r$ and $F^F_r$ will be on board late for their next operating sectors, $F_s$ and $F_t$, respectively, i.e. both flights are delayed on their departure.

*Unit Crewing* is an approach to construct crew schedules so that members of a crew perform the same sequence of flights for as much of their duty periods as possible. A unit crewed schedule is considered to be more operationally robust in the sense that if members of a crew perform the sequence of flights for as much of their duty periods as possible, it is less likely to delay other flights due to waiting for a member of the crew from a disrupted upstream flight.

A *unit crewing connection* is a connection between two consecutive flights, $F_r$ and $F_s$, where each member of the crew for $F_r$, operates $F_s$ as a subsequence. Any connection between the start of a duty period and the first operating sector in the duty period is also considered to be a unit crewing connection.

If some crew members operate flights $F_r$ and $F_s$ as a subsequence and some crew members operate $F_s$ as a starting sector of their duty period, delay might occur on $F_s$ if $F_r$ is disrupted, as $F_s$ needs to wait for the crew members from $F_r$. So if each crew member operates the sector $F_s$ as a starting sector in their duty period, then that is a unit crewing connection between the start of duty and $F_r$.

However, connections between the last operating sector in the duty period and the end of that duty period is never considered as a unit crewing connection, as no downstream flight departure time is affected.

It is usual to obtain a crew schedule for one rank in terms of minimal cost (i.e. the traditional ToD planning model in GSPP formulation) and then select as many unit crewing connections as possible when solving the next rank. In this paper, we addressed the result obtained from unit crewing Captains to the minimal cost schedule for the First Officers.

### 3.1 Bi-Criteria Model

In practice, it is unlikely to obtain a completely unit crewed ToD solution, because of the differences in operating rules for each rank and differences in the balance of each crew base for each rank. There is always a trade-off between minimizing the crew cost and maximizing the number of unit crewing connections. A cost minimal crew schedule may have a small number of unit crewing connections, while many unit crewing connections may results in high crew cost.

Therefore, a new objective is required to maximize the number of unit crewing connections as well as maintaining the economy of the crew schedule. This results a bi-criteria ToD planning optimization model, which is an extension of the GSPP formulation:

\[
\begin{align*}
\text{Minimise} & \quad z_1 = c^T x \\
\text{Minimise} & \quad z_2 = \tilde{u}^T x \\
\text{subject to} & \quad Ax = e & (\text{bi-GSPP}) \\
& \quad Mx \begin{cases} 
\leq b \\
\geq b 
\end{cases} \\
& \quad x \in \{0, 1\}^n
\end{align*}
\]

where $\tilde{u}_j$ is the number of non-unit crewing connections in the $j^{th}$ ToD.
3.2 Penalty Method

In multi-criteria optimization problems, there will usually exist several, often many Pareto optimal solutions, all with different crew cost and number of unit crewing connections. One way to solve the bi-criteria GSPP robust ToD planning problem is the weighted sum method, which consists in minimizing a convex combination of the objectives, so that Pareto optimal crew schedules can be found by carefully selecting different weights on the objective.

Since this method simply consists in adding a penalty on the planned crew cost, we called this method the Penalty Method, the unit crewed ToD planning problem is reformulated as follow:

\[
\text{Minimise} \quad z = (c + p\tilde{u})^T x \\
\text{subject to} \quad Ax \leq e \quad (bi-p-GSPP) \\
Mx = b \\
x \in \{0, 1\}^n
\]

where \( \tilde{u} \) is the number of non-matched unit crewing connection in the \( j^{th} \) ToD, while \( p \) is the penalty applied to the cost objective of each additional non-matched unit crewing connection found in the \( j^{th} \) ToD, i.e. \( p\tilde{u} \) is the total penalty applied to the planned crew cost \( c_j \) for the \( j^{th} \) ToD. Figure 2 shows the results obtained from different penalty values.

Although the penalty method yields a problem of the form of GSPP, and the optimal solution is known to be Pareto optimal, there are serious drawbacks.

First of all there is no interpretation of the sum of planned cost (in dollars) and the weighted penalty on the non-unit crewing connections. Secondly, the specification of the penalty is a difficult task, in particular because the optimal solution will be very sensitive to slight changes of the penalty, as there are a large number of feasible solutions with similar cost, but only a few Pareto optimal solutions may be found this way.

Non-Unit Crewing Objective (\( u^T x \))

![Figure 2: Relationship between Cost and Non-Unit Crewing Objectives](image)
3.3 Elastic Constraint Method

We also implemented the elastic constraint method introduced by Ehrgott and Ryan that is able to generate all Pareto optimal schedules, but is also flexible enough to allow management to select the efficient solution they will eventually implement.

This technique is based on the idea of only optimizing one of the objectives, while transforming the other(s) into constraint(s), specifying an upper bound $\varepsilon$ on their values. It is well known that by varying the value of $\varepsilon$, all Pareto optimal solutions can be found.

Besides being able to generate all Pareto optimal solutions by varying the upper bounds on the objective constraints, it allows an easy interpretation of its parameters: Management could simply specify the additional crew cost they are willing to concede in order to improve unit crewing. That is, the additional crew cost is treated as the opportunity cost for the additional unit crewing connections.

For the unit crewed ToD planning problem, we obtain the following reformulation of the bi-criteria problem ($bi$-GSPP).

Minimise $z = \bar{u}^T x + ps_u$

subject to $Ax \leq e$

$Mx = b$

$c^T x + s_l - s_u = \varepsilon$

$x \in \{0, 1\}^n$

$s_l, s_u \geq 0$

The additional constraint derived from the cost objective function is an elastic constraint. We use $\varepsilon$ as an upper bound on the cost objective, which depends on the optimal objective value $c_{opt}$ of the LP relaxation obtained from solving the GSPP with the cost objective alone, i.e. $\varepsilon = (1 + oc / 100) c_{LP}$. The parameter $oc$ specifies the desired percentage increase in the cost objective from the optimal LP relaxation that management might be prepared to consider.

Figure 3: Integer Solutions Obtained from Elastic Constraint Method
The introduction of the cost surplus $s_u$ allows small violation of the cost constraint to avoid the computational difficulties that arise by the newly introduced knapsack type constraint. A penalty term $p$ is added for the surplus variable $s_u$ in the objective function.

Figure 3 shows the results obtained from varying the penalty of the cost surplus and $o_c$ value form 0 to 16 in increment of 1.

4 Unit Crew Branching

The solutions obtained from either the penalty method or the elastic constraint method are proven to be Pareto optimal (see Ehrgott and Ryan, 2002), but only if we consider the crew schedule that has been unit crewed. That is, if we look at the crew schedules for both ranks, the solutions might not be Pareto optimal, and by solving both ranks simultaneously rather than in sequence it might be possible to improve both cost and degree of unit crewing.

This is because before we solve the unit crew ToD planning problem, a cost optimized crew schedule for one rank needs to be found. When we construct the cost minimal crew schedule for this rank, connections in the ToDs were selected by considering the cost and feasibility of the schedule for this rank only, without considering the optimality (or even feasibility) for the other rank.

The only way to find ToD solutions for different crew groups that are unit crewed with each other is to solve the ToDs planning problem for all rank simultaneously, so that during the process of ToD generation, consecutive flight connections that are preferable for all ranks can be constructed. We solve the following combined model and use a special strategy to select as many unit crewing connections as possible during the branch and price process.

Minimise $z = c_1^T x_1 + c_2^T x_2$

subject to

$A_1 x_1 = e$

$M_1 x_1 \leq b_1$

$A_2 x_2 = e$ (2-GSPP)

$M_2 x_2 \leq b_2$

$x_1, \quad x_2 \in \{0, 1\}^n$

Unit crew branching uses a special flight pair selection and branching strategy to ensure the ToDs generated from both ranks to be unit crewed as much as possible.

The selection strategy is based on the simple observation of the definition of unit crewing connections, which is a set of consecutive flight sectors that can be operated as a subsequence for both crew ranks, and therefore, it is a proper subset of the set of follow-on flights. That is, if a pair of flights ($F_r$ and flight $F_s$) is able to be unit crewed with $F_s$ following $F_r$ in the schedule, this flight pair ($F_r$ and $F_s$) must be a follow-on sector pair.

The idea of unit crew branching is that a connection is unit crewed if both crew ranks are forced to perform the same connection between a pair of consecutive flights. Alternatively, if both ranks are banned to perform the same connection between two flight sectors, then they will perform an alternative flight sequence. The alternative flight sequence will then be forced or banned.

By using unit crew branching, the number of unit crewing connections increased by
2.8% in comparison to the result obtained from solving two ToD planning problems independently without consideration of unit crewing, but the property of cost optimality is retained. However, this method does not allow an analysis of the trade off between cost and unit crewing.

5 Following-the-Aircraft Objective

Instead of the unit crewing objective we introduce a new objective into the 2-GSPP model and relax the cost constraint to become the elastic constraint. This objective must, however, be meaningful and correlated to both of the cost objective and the unit crewing objective. If this objective is not correlated to the crew cost, the relaxed cost constraint is not effective in the model, and if this objective is not correlated to the unit crewing objective, effective unit crew branching cannot be imposed.

The number of connections following the same aircraft, which is a derivative from the robustness measure that used by Ehrgott and Ryan is a good choice. If the number of connections following the same aircraft increases, the cost increases, and if the crew perform the same connection that is following the same aircraft, then this connection is unit crewed automatically. Furthermore, if more connections following the same aircraft are contained in the solution, the crew schedule becomes more robust.

After the new objective is introduced, the cost objective is relaxed to an elastic constraint form, and unit crew branching is used to select as many number of unit crewing connections as possible.

Maximise $z = \hat{a}_1^T x_1 + \hat{a}_2^T x_2$
subject to
$A_1 x_1 = e$
$M_1 x_1 \leq b_1$
$A_2 x_2 = e$
$M_2 x_2 \leq b_2$
$c_1^T x_1 + c_2^T x_2 + s_l - s_u = \varepsilon$
$x_1, x_2 \in \{0, 1\}^n$
$s_l, s_u \geq 0$

where elements of the vectors $\hat{a}_1$ and $\hat{a}_2$ represent the number of connections following the same aircraft in each ToD for each crew rank. The value of $\varepsilon$ is an upper bound on the cost objective, which depends on the sum of optimal objective values $c_{LP1}$ and $c_{LP2}$ from the LP relaxation obtained from solving the two independent ToD planning problems for the crew ranks in GSPP formulation and $o_c$, the desired percentage increase in the cost objective from the optimal LP relaxation that management might be prepared to consider, i.e. $\varepsilon = (1 + o_c / 100) (c_{LP1} + c_{LP2})$.

Figure 4 shows the interaction between the three objectives. The horizontal and vertical axes represent the percentage changes in the cost and following-the-aircraft objectives in comparison with the cost minimal solution from solving the two ToD planning problems independently. The areas of the circles represent the percentage changes in unit crewing objective in comparison with the result obtained from solving the two ToD planning problems independently with crew cost as the objective.

Although the combined ToD planning problem in 2-EC-GSPP formulation with addition of the use of unit crew branching might results in a more robust crew schedule
in terms of unit crewing and following-the-aircraft, but there is a problem to find a crew schedule that maximize the number of unit crewing connections only with a desired amount of crew cost. This is because the use of unit crew branching strategy is only able to select the maximum number of unit crewing connections from the two sets of ToDs as long as the following-the-aircraft objective and cost objective is optimized.

Percentage Change on Number of Connections Following-the-Aircraft

![Figure 4: Trade-Off between Crew Cost, Following-the-Aircraft and Unit Crewing Objectives](image)

6 Conclusion

Our results clearly show that more unit crewed schedules can be generated by conceding only small increases in cost of operation. The proposed methodology can be adapted in current optimization systems for crew scheduling, and once staff is familiar with the additional parameters, should require little more effort in constructing more robust schedules than schedule construction takes today.

The unit crewing objective can be improved by solving a combined model simultaneously with coupling constraints to link the connections of both ranks.

References


