Market Analysis illustrated by the Mandelbrot Model

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Abstract

We consider global order of packaged goods market. First, we show global order exists there. Namely, the relationship between ranking and size of product obeys Mandelbrot model. This means a market is illustrated by both the rate of new entry of the product into the market and the expected growth rate of the product. Next, based on this we classify the market under four, that is, the inertia market, the newfangled market, the flourishing market and the stagnant market.

1 Introduction

Structure obeying specific order has been observed in various organizations and groups, and then their global character has been discussed. For example, the relationship between ranking and the population of cities obeys the power law (the Zipf law). Namely, it is as follows:

$$s(r) = br^{-a}, \quad (1)$$

where

r: ranking of city,
$g(r)$: the population of the r-th ranking city.

We can observe this relationship in various periods, countries and areas [2] [11] [5].

Another relationship, frequency distribution derived from equation (1), is as follows (the Lotka law):

$$f(n) = dn^{-c}, \quad (2)$$

where

n: the number of papers,
$f(n)$: the number of chemists publishing n papers.

Moreover, various numerical expressions have been formalized by attention to the power law [3] [10], and many phenomena obeying these laws have been reported [1]. For example,

- ranking and a frequency of personal income within a nation,
- ranking and a frequency of words in an English work,
- ranking and a frequency of alphabets in English,

・ the amounts of import to (export from) a nation and ranking by nations in a year and so on.

Social phenomena obeying the power law were paid attention to and discussed especially in the fields of urban economy and bibliometrics, and many excellent studies and practical implications have been showed. However, market phenomena rarely have been paid attention by the view of the power law. In this field, there are only few studies that discussed the firms' distribution [12] [4].

The essence of the power law is to describe skew. Namely, it is to observe phenomena obeying skew distribution, to formalize them, and to discuss the mechanism from which how they emerge. We can point out skew not only in the fields of urban economy and bibliometrics but also in various market phenomena concerning consumers and firms. For example, the bipolar concentrated market structure is formed. Namely, few big firms, brands, standards and products have huge market share and many small firms, brands, standards and products have only a small market share. That is, their distributions are highly skewed [8]. Moreover, firms' marketing effort which is measured by advertising expenditure to each product is also highly skewed [6] [7]. Hence, we may apply the power law to market phenomena concerning consumers and firms.

In this study, first we will show the global order that the relationship between the ranking and the size of products obeys the power law in market (the 1st subject). Then, if we can observe it, we discuss the implication of it (the 2nd subject).

In this study, the object is limited to packaged goods bought in household block. We use product, namely stock keeping unit (SKU), for our analysis. We will focus on market structure itself and discuss its order; on the other hand, we will not refer to an individual product that forms its market.

2 Methods and Materials

2.1 Model

There are two types of equation in the power law, the first is an equation expressed by rank-size, the second expressed by frequency distribution. As both are equivalent mathematically, we consider only the former in this study. There are two types of expression of rank-size; one is the Zipf model, equation (1), and another is Mandelbrot model, which is the general expression of the Zipf model.

[8] [9] discussed global order of market by attention to the power law. These studies showed the relationship between ranking and size of product obeys the Zipf law by using purchased data gathered from packaged goods markets. Here, we discuss the relationship between ranking and size of product by applying another power law, the Mandelbrot model. Namely,

$$s(r)=b(r+k)^{-a}, \quad (2)$$

where

r: ranking of product sales,

$s(r)$: sales volume of the r-th ranking product,

n: the number of products in a market,

$0<a$, b and k.

$1 \leq r \leq n$

By adding parameter k ($0<k$) to the Zipf model and by $1 \le r \le N$, we postulate explicitly that the size of products is finite and we avoid that equation (2) diverges. We estimate our parameters and examine how data fits into equation (2) by using home scanning panel data gathered from packaged goods markets. Namely, a, b and k are estimated to minimize the sum of the residuals.

2.2 Data

We use home scanning panel data (Quick Purchase Report) gathered by Tokyu Agency Inc. and Tokyu Research Institute in this study, and

- area: cities, towns and villages within 30 km. from Tokyo station in Japan,
- object: households in which the age of the housewife is under 59,
- sampling method: random two stage sampling,
- sample size: 2,500 households.

We gather data relating to 13 markets. Namely, these markets are as follows.

- Food and Drink: sauce, catsup, butter, margarine and fastspread, instant curry, cooked curry, instant coffee, cola, 100% pure fruit juice and beer,
- household Commodities: heavy detergent, shampoo and tin foil.

Data from Jan. 1 1990 to Dec. 31 1999 is added up for one year respectively. That is, we examine 13 markets * 10 years = 130 cases. We consider size of product as sales volume, and we use purchasing dollar volume per 100 households.

3 Result

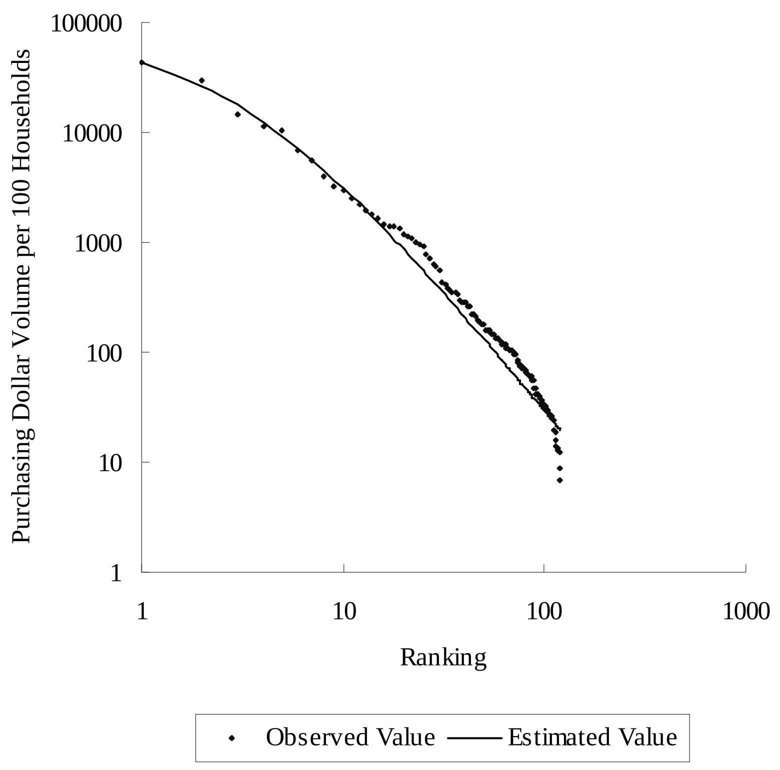

Figure 1. Ranking and Size of Product by Mandelbrot Model
(Heavy Detergent, 1998)

Equation (2) was investigated by the above way. We showed here the result of the case of the heavy detergent market among others. We adapted data in 1998 to equation (2). At this time an equation was estimated below (figure 1):

$$s(r) = 1{,}046{,}259(r+3.086)^{-2.261}$$

where

r: the ranking of product sales.

$s(r)$: purchasing dollar volume per 100 households of the r-th ranking product,

The sample size was 120, the standard deviation of the residuals was 478, and the coefficient of determination was $R^2=0.991$. Since the minimum and the maximum value of response variable were 42,543 and 6.9 respectively, it could be evaluated that the standard of deviation of the residuals was sufficiently small. Moreover, the coefficient of determination was also good. Thus, equation (2) was fit for data.

When we observed 13 packaged goods markets from 1990 and 1999, both the standard deviation of the residuals and the coefficient of determination were good in all markets, in all periods (table 1). Thus, equation (2) fitted the data. Hence, we showed there is global order in packaged goods market. Namely, the relationship between ranking and size of product obeys equation (2).

4 Discussion

4.1 Implication of parameter *a*

When equation (2) holds in market, we discuss the implication of it. Suppose the ranking of the products are r_1, r_2 ($r_1<r_2$) and the size of the products are s_1, s_2 ($s_1>s_2$), equation (2) is expressed as follows:

$$\frac{s_1}{s_2} = \left(\frac{r_2+k}{r_1+k}\right)^a . \quad (3)$$

As $(r_2+k)/(r_1+k) >1$, supposing this is constant, equation (3) becomes larger as a becomes larger. That is, the ratio of higher-ranked products to lower-ranked products (S_1/S_2) becomes larger as a becomes larger. Hence, we can understand that a is the indicator to measure the concentration and dispersion of the market share.

Furthermore, it is reasonable to suppose that a shows the rate of new entry of the product into market when we quote [12]. Namely, the rate of new entry of the product is low when a is large, and it is high when a is small.

4.2 Implication of parameter *k*

According to [4], the Gibrat's law of proportionate effect acts when the Zipt model holds. That is, if k would be 0 in equation (2), the Gibrat's law of proportionate effect acts. At this time, the expected percentage changes for the totality of components in a size group are independent of size. In other word, it means that products belonging to the same size (sales volume) group earn new sales in proportion to the total size (sales volume) of their group. That is,

Table 1. Regression Analysis (Extract)

Market	*Year*	*Parameter*			*Sample Size*	*Standard* Deviation *of Residuals*	*Coefficient of Determination*	*Range of Response Variable*	
		a	*b*	*k*					
sauce	1990	1.343	30,411	0.421	137	196	0.990	19,093	2.7
	1991	1.445	46,764	0.803	150	215	0.988	20,283	2.1
	1992	1.412	39,441	0.607	157	184	0.991	20,371	5.5
	1993	1.419	41,180	0.563	150	187	0.992	22,087	5.5
	1994	1.444	39,902	0.517	139	173	0.994	22,027	2.8
	1995	1.270	22,364	0.055	130	226	0.988	21,058	7.1
	1996	1.590	49,134	0.874	156	150	0.993	18,238	3.5
	1997	1.556	48,739	0.841	149	140	0.994	18,997	6.2
	1998	1.398	29,916	0.481	155	177	0.988	17,445	0.7
	1999	1.248	18,475	0.028	159	141	0.992	17,965	1.9
catsup	1990	1.800	73,374	0.692	82	346	0.990	28,261	9.0
	1991	1.467	41,539	0.305	88	225	0.996	28,155	6.9
	1992	1.570	52,323	0.577	83	313	0.991	25,484	6.6
	1993	1.693	72,914	0.827	76	134	0.999	26,286	8.8
	1994	2.039	107,646	1.046	80	243	0.994	24,851	6.9
	1995	1.499	24,293	0.000	93	267	0.990	24,723	4.7
	1996	1.494	23,599	0.000	94	113	0.998	23,676	9.0
	1997	1.550	24,908	0.000	84	205	0.995	24,920	6.5
	1998	1.423	21,659	0.000	87	213	0.993	21,663	6.9
	1999	1.493	24,571	0.142	94	309	0.982	19,997	5.6
butter	1990	1.643	74,426	0.716	42	483	0.992	30,848	14.3
	1991	1.291	30,446	0.001	42	695	0.982	30,703	10.6
	1992	1.661	90,017	0.975	38	1073	0.961	29,498	10.4
	1993	1.326	31,114	0.001	43	739	0.980	31,776	6.9
	1994	1.335	34,984	0.330	53	458	0.985	24,199	13.2
	1995	1.334	34,162	0.239	55	513	0.983	25,994	10.7
	1996	1.789	150,513	1.991	66	216	0.996	21,421	13.2
	1997	1.918	171,667	1.825	51	405	0.990	23,542	13.8
	1998	1.831	139,507	1.777	49	370	0.990	21,664	3.4
	1999	1.427	35,366	0.320	52	250	0.995	23,874	16.1
margarine	1990	1.513	96,963	1.335	87	717	0.961	25,811	9.0
and fastspread	1991	1.249	69,444	1.161	84	599	0.975	27,606	8.0
	1992	1.172	50,043	0.633	78	535	0.982	28,797	3.7
	1993	1.331	68,568	1.066	85	424	0.986	26,711	4.1
	1994	1.513	96,963	1.335	76	381	0.990	27,197	8.7
	1995	1.339	51,972	0.757	63	261	0.995	24,602	9.8
	1996	1.121	26,473	0.066	69	379	0.988	24,910	3.5
	1997	1.287	41,618	0.462	62	283	0.994	25,618	7.2
	1998	1.213	35,003	0.354	69	328	0.991	24,449	10.9
	1999	1.148	27,171	0.131	67	402	0.986	23,858	6.3
instant curry	1990	1.337	189,774	4.536	251	228	0.989	19,767	8.0
	1991	1.071	63,595	1.242	263	337	0.982	28,125	4.3
	1992	1.122	79,992	1.641	233	407	0.978	28,672	6.5
	1993	1.399	218,507	3.891	237	260	0.990	25,312	6.9
	1994	1.754	785,556	6.614	225	236	0.992	21,871	6.9
	1995	1.887	1,432,451	8.878	221	149	0.996	18,323	7.0
	1996	1.876	1,591,324	10.326	219	236	0.989	16,257	6.3
	1997	1.957	1,978,578	9.993	217	253	0.989	16,853	7.2
	1998	1.901	1,561,620	9.900	181	265	0.988	16,628	6.9
	1999	1.671	537,587	6.669	179	262	0.988	17,158	6.4

Table 1 (continued). Regression Analysis (Extract)

Market	*Year*	*Parameter* *a*	*b*	*k*	*Sample Size*	*Standard* Deviation *of Residuals*	*Coefficient of Determination*	*Range of Response Variable*	
instant coffee	1990	1.362	67,536	0.000	124	481	0.995	68,428	6.1
	1991	1.202	60,535	0.019	128	716	0.986	59,773	6.9
	1992	1.214	63,073	0.000	126	446	0.995	63,371	4.0
	1993	1.684	228,113	1.236	137	275	0.998	58,842	6.5
	1994	1.484	120,857	0.621	135	544	0.992	59,344	7.0
	1995	1.641	219,388	1.524	163	601	0.984	48,428	7.1
	1996	2.249	1,017,609	2.982	154	525	0.988	45,598	3.5
	1997	1.904	351,178	1.921	152	341	0.995	44,886	3.3
	1998	2.068	891,814	2.994	147	740	0.982	49,031	6.9
	1999	2.015	734,074	2.767	162	700	0.982	48,704	5.8
cola	1990	1.462	41,229	0.000	34	1119	0.978	42,247	62.1
	1991	3.785	32,008,713	4.823	46	943	0.982	41,412	4.0
	1992	1.657	132,301	1.391	57	488	0.990	31,593	7.0
	1993	1.990	236,995	2.058	58	664	0.973	24,965	3.4
	1994	1.193	30,230	0.000	78	355	0.991	30,724	4.8
	1995	1.516	69,831	0.718	85	247	0.996	30,652	7.0
	1996	1.487	51,673	0.331	86	147	0.999	33,759	3.9
	1997	1.409	41,495	0.000	70	695	0.983	42,361	5.0
	1998	1.630	46,404	0.000	62	936	0.977	47,432	5.2
	1999	1.467	41,254	0.000	52	899	0.979	42,247	3.1
heavy detergent	1990	1.325	84,794	0.001	130	648	0.994	85,861	2.7
	1991	1.188	80,864	0.001	137	766	0.991	82,187	8.2
	1992	1.442	172,571	0.841	141	434	0.997	71,571	11.9
	1993	1.483	193,935	1.184	122	568	0.993	61,356	13.6
	1994	1.279	88,586	0.415	113	432	0.995	57,010	13.9
	1995	1.815	576,913	4.063	136	609	0.976	30,578	1.4
	1996	1.158	52,690	0.455	153	620	0.969	33,706	3.4
	1997	1.931	373,856	1.991	128	699	0.981	44,115	7.2
	1998	2.261	1,046,259	3.086	120	478	0.991	42,542	6.9
	1999	2.802	7,616,378	5.876	125	813	0.964	31,549	6.4
shampoo	1990	1.004	62,156	2.546	435	151	0.991	17,793	7.2
	1991	0.914	41,888	1.238	481	240	0.976	21,511	5.5
	1992	1.267	194,102	7.261	512	83	0.996	12,948	5.3
	1993	0.829	27,192	0.437	501	246	0.970	21,391	6.7
	1994	1.140	97,959	2.925	421	153	0.993	21,461	7.0
	1995	1.019	63,707	3.171	475	110	0.993	15,195	7.1
	1996	1.044	59,589	1.995	493	172	0.986	17,960	7.0
	1997	0.970	49,091	1.791	482	154	0.989	18,627	6.6
	1998	0.934	40,870	2.276	495	157	0.982	14,774	6.1
	1999	0.730	17,011	0.001	573	236	0.954	18,188	6.4
tin foil	1990	2.150	139,776	3.040	81	68	0.995	7,008	8.9
	1991	1.490	18,713	0.869	86	102	0.989	7,452	3.4
	1992	1.908	45,996	1.571	88	112	0.987	7,490	3.3
	1993	1.933	52,889	1.874	85	134	0.979	6,765	6.2
	1994	1.423	10,743	0.275	75	46	0.998	7,606	6.2
	1995	2.463	213,397	3.672	64	156	0.959	4,588	6.3
	1996	2.476	283,040	4.390	71	159	0.948	4,118	4.8
	1997	2.327	164,108	3.804	64	123	0.969	4,123	5.5
	1998	2.176	88,358	3.092	67	76	0.986	3,998	6.1
	1999	1.778	22,417	1.796	71	68	0.984	3,566	3.7

$$\Delta S_r = \frac{S_r}{\sum_{r=1}^{n} S_r} \sum_{r=1}^{n} \Delta S_r ,$$

where

S_r : the total size of products belonging to the r-th ranking group,

n: the number of product groups in a market.

At this time, equation (2) becomes a straight line on a logarithmic graph (figure 2).

Next, we discuss a case of k>0. For all r, $b(r+k)^{-a}<br^{-a}$. The ratio of the left side to the right side becomes small as r becomes small (the product size becomes large). That is, as the ranking becomes higher, the product size that is derived from equation (2) becomes smaller than the product size that is derived from the Zipf model, in which the Gibrat's law of proportionate effect acts. At this time, equation (2) becomes concave curve (figure 2).

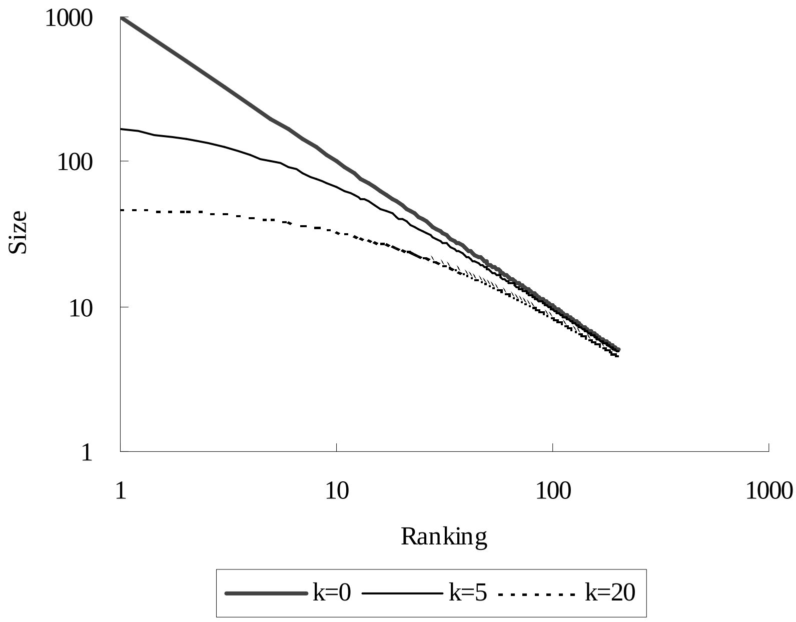

Figure 2.

Change of Parameter k in Mandelbrot Model (a =1)

4.3 Empirical validity

Above discussions are reasonable empirically, because we can interrupt a market character precisely as follows by attention to both the rate of new entry of the product and the expected growth rate of the product. For example, the top 10% of products dominated 92% market share in the cola market in 1998 in Japan. On the other hand, the top 20% of products dominated 82% market share in the 100% pure fruit juice market in 1998. We can analyze that the reason of this difference is that their expected growth rates of the higher-ranked products are different. That is, the values of k are 0.001 in the cola market and 23.812 in the 100% pure fruit juice market.

Moreover, we can analyze a change of a market by using both the rate of new entry of the product and the expected growth rate of it. For example, in the heavy detergent market in 1996 and 1997, the top 20% of products dominated 88% and 89% market share respectively and the top 30% of products dominated 93% overall. Did no change happen in the market in those periods? Actually, the market changed. New marketable products were launched in 1995 and these products have occupied higher-ranked positions in 1997. Hence, the expected growth rate of the higher-ranked products became larger (the value of k changed from 4.063 in 1995 to 1.991 in 1997). On the other hand, the rate of new entry of the product declined (the value of a changed from 1.815 in 1995 to 1.931 in 1997). Therefore, the number of the products decreased from 136 in 1995 to 128 in 1997, and the concentration ratio became higher.

5 Implication

We suggest our implication for marketing strategy based on interpretation of both a and k. Markets can be classified into four under the values of parameters. It is reasonable to interpret the character of each market as follows (figure 3).

- The Inertial (Conservative) Market. The rate of new entry of the product is low and the expected growth rate of the higher-ranked products is relatively high. Hence, market share is concentrated.
- The Newfangled (Throwaway) Market. The rate of new entry of the product is high and the expected growth rate of the higher-ranked products is relatively low. Hence, market share is dispersed.
- The Mature (Stagnant) Market. The rate of new entry of the product is low and the expected growth rate of the higher-ranked products is relatively low. Because the higher-ranked products are not charming, nevertheless new products are hard to emerge, market share is not dispersed.

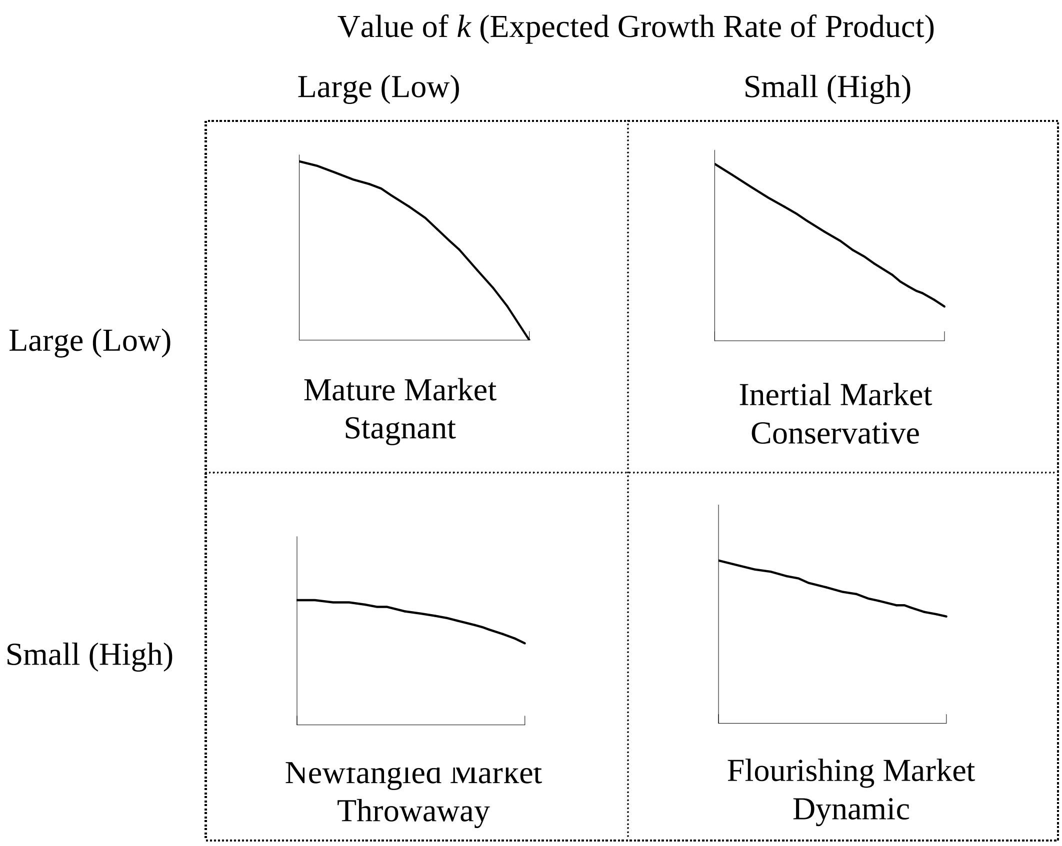

Figure 3. Market Classified by Mandelbrot Model

- The Flourishing (Dynamic) Market. The rate of new entry of the product is high and the expected growth rate of the higher-ranked products is relatively high. Because new products emerge lively, nevertheless the higher-ranked products are charming; market share is not concentrated.

We show examples of the market classification as follows. We can enumerate the cooked curry market in 1990 as the example of a mature market, and this market in 1998 as the example of a flourishing market. Size of this market was small in 1990 (purchasing dollar volume per 100 households of this market was 64,592 Japanese Yen, that of the instant curry market, which competes with the cooked curry market, was 227,478 JYN). At this time, new products were not launched lively (the number of products was 105 in the cooked curry market while it was 251 in the instant curry market). The expected growth rate of the higher-ranked products would be relatively low because of the lack of marketable products and because of finiteness of the market. Hence, it is reasonable to suppose that the mature market formed (figure 3).

After that, the cooked curry market grew by acquiring demand switched from the instant curry market (purchasing dollar volume per 100 households grew from 64,592 in 1990 into 102,532 JYN in 1998). The expected growth rate of the higher-ranked products would increase because marketable products were launched and because the total market expanded. However, the concentration ratio declined because many new products were launched (the number of product increased from 105 in 1990 to 286 in 1998). Therefore, it is reasonable to suppose that the flourishing market formed.

We classified markets based on both the rate of new entry of the product and the expected growth rate of it. Such trials suggest useful implication to marketing strategy. For example, we obtain our implication concerning portfolio strategy by comparing the characters of several markets.

6 Conclusion

In this study, we showed global order exists in packaged goods market. Namely, the relationship between ranking and size of product obeys Mandelbrot model. This means market is illustrated by both the rate of new entry of the product into the market and the expected growth rate of the product. Next, based on this we classified the market under four and we showed this trial was valid empirically by enumerating concrete cases.

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